

EDUZEDCO
121/2 MATHEMATICS
Paper 2
Form 3 End Term Examination

Time: 2 ½ Hours

Name: Adm No:
 School: Class:
 Signature: Date:

INSTRUCTIONS TO CANDIDATES

- (a) Write your name and index number in the spaces provided at the top of this page.
- (b) Write your school name, sign and write the date of the examination in the spaces provided above.
- (c) This paper consists of **Two** sections: **Section I** and **Section II**.
- (d) Answer **ALL** questions in Section I and any **five questions** from Section II.
- (e) Marks may be given for correct working, relevant points, and clear presentation.
- (f) Non-programmable silent electronic calculators may be used where necessary.
- (g) KNEC Mathematical Tables may be used.
- (h) This paper consists of **8 printed pages**.
- (i) Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.
- (j) Candidates should answer the questions in English.

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total	Grand Total

SECTION I

1. Given that y varies directly as x and $y = 15$ when $x = 5$, find the value of y when $x = 9$.
2. Solve for x : $2^{2x-1} = 32$.
3. Simplify: $\log_{10} 1000 + \log_{10} 0.01$.
4. Write 0.00256 in standard form, then find its reciprocal. Give your answer in standard form.
5. Rationalise the denominator and simplify: $\frac{1}{\sqrt{3}}$.
6. The third term of a geometric progression is 24 and the fifth term is 96. Find the common ratio (assume positive).

7. Solve the quadratic equation $x^2 - 4x - 5 = 0$ by completing the square.

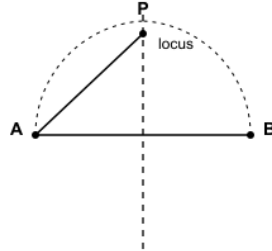
8. Find the determinant of matrix $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$.

9. Use the binomial expansion to find the coefficient of x^3 in the expansion of $(2 - x)^6$.

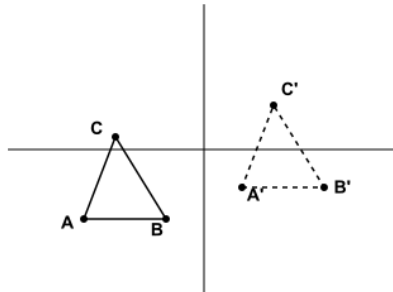
10. y varies directly as the square of x and inversely as z . When $x = 2$ and $z = 4$, $y = 3$. Find y when $x = 4$ and $z = 8$.

11. Without using a calculator, find the value of $\sin 150^\circ + \cos 240^\circ$.

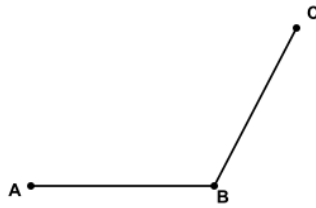
12. The locus of a point P is given by $PA = PB$ where $A(1, 2)$ and $B(3, 4)$. The locus of P also satisfies the condition that P lies on the line $x = 2$. Find the coordinates of the points of intersection of these two loci.



13. A transformation is represented by the matrix $M = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$. Find the image of the point $P(2, -1)$ under this transformation.



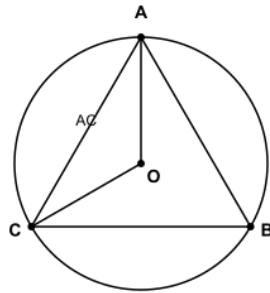
14. Given vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, find:



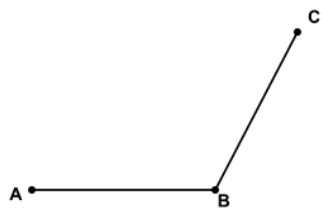
(a) Find $|\mathbf{a} + 2\mathbf{b}|$. [2 marks]

(b) Find the unit vector in the direction of $\mathbf{a} - \mathbf{b}$. [2 marks]

15. In the diagram, AC is a diameter of the circle with centre O . $\angle ABC = 70^\circ$. Find $\angle ACB$.



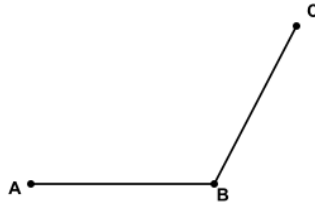
16. Find the scalar product of vectors $\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$.



SECTION II

Answer any five questions from this section.

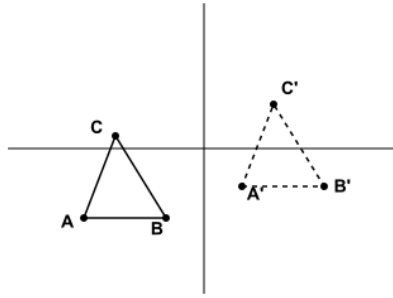
17. The position vectors of points A, B and C are $\vec{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\vec{b} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\vec{c} = 2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$ respectively. Point D divides AB in the ratio 2 : 3 internally. Point E divides AC in the ratio 3 : 1 externally. [10 marks]



(a) Express the position vectors of D and E in terms of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$.

(b) Determine the vector $\vec{DE}$ and its magnitude, correct to 3 significant figures.

18. A triangle with vertices $P(1, 2)$, $Q(3, 1)$ and $R(2, 4)$ undergoes two successive transformations. First, a reflection in the line $y = x$ represented by matrix $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, followed by a rotation through 90° anti-clockwise about the origin represented by matrix $B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [10 marks]

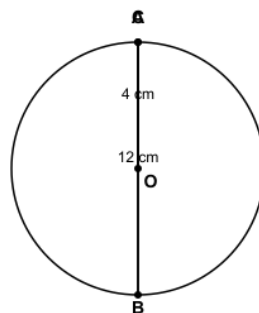


(a) Determine the single transformation matrix that represents the combined transformation.

(b) Find the coordinates of the image of triangle PQR under the combined transformation.

(c) If the combined transformation is applied to a point S and its image is $S'(3, 1)$, find the coordinates of S .

19. In the diagram below, O is the centre of the circle. AB is a chord of length 12 cm. OC is perpendicular to AB and meets AB at C . Given that $OC = 4$ cm, [10 marks]



(a) find the radius of the circle.

(b) If the chord AB is extended to D such that $BD = 6$ cm and AD is a tangent to the circle at A , find the length of AD .

20. [10 marks]

(a) Expand $(1 + 3x)^5$ in ascending powers of x up to the term in x^3 .

(b) Use your expansion to estimate the value of $(1.03)^5$ correct to 4 decimal places.

21. The population of a town is increasing at a rate of 8% per annum. The current population is 250,000. [10 marks]

(a) What will be the population after 3 years?

(b) After how many years will the population first exceed 400,000? (Use logarithms.)

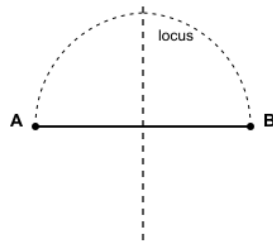
22. A sum of money invested at compound interest amounts to sh 13, 310 after 3 years and sh 14, 641 after 4 years. [10 marks]

(a) Find the rate of interest per annum.

(b) Determine the initial principal.

(c) How much will the investment amount to after 6 years?

23. [10 marks]



Find the equation of the locus.

Describe the locus and sketch it.

24. A quadratic equation $x^2 + kx + 9 = 0$ has two equal roots. [10 marks]

(a) Determine the possible values of k .

(b) For each value of k , write the resulting quadratic expression in the form $(x + a)^2$ and solve the equation.

(c) If the quadratic expression $x^2 + kx + 9$ is a perfect square, find the value of k and factorise the expression completely.

121/2 MATHEMATICS MARKING SCHEME

Form 3 Paper 2

SECTION I (50 marks)

1. Given that y varies directly as x and $y = 15$ when $x = 5$, find the value of y when $x = 9$. (3 marks)

M1: Form the equation $y = kx$ [1]

M1: Substitute given values to find k : $15 = k(5) \rightarrow k = 3$ [1]

A1: $y = 3(9) = 27$ [1]

2. Solve for x : $2^{2x-1} = 32$. (4 marks)

M1: Express 32 as 2^5 [1]

M1: Equate exponents: $2x - 1 = 5$ [1]

A1: Solve: $2x = 6 \rightarrow x = 3$ [1]

B1: Correct final answer stated [1]

Alternative method/answer:

Alternative: Taking logs: $(2x-1)\log 2 = \log 32 \rightarrow 2x-1 = 5 \rightarrow x=3$ (M1 for logs, A1 for solution)

3. Simplify: $\log_{10} 1000 + \log_{10} 0.01$. (4 marks)

M1: $\log_{10} 1000 = \log_{10} 10^3 = 3$ [1]

M1: $\log_{10} 0.01 = \log_{10} 10^{-2} = -2$ [1]

A1: Sum = $3 + (-2) = 1$ [1]

B1: Correct final answer [1]

4. Write 0.00256 in standard form, then find its reciprocal. Give your answer in standard form. (3 marks)

M1: Standard form: 2.56×10^{-3} [1]

M1: Reciprocal = $1/(2.56 \times 10^{-3}) = (1/2.56) \times 10^3$ [1]

A1: $= 3.90625 \times 10^2$ (accept 3.906×10^2 or 390.625) [1]

Alternative method/answer:

Accept 390.625 in standard form as 3.90625×10^2

5. Rationalise the denominator and simplify: $(1)/(\sqrt{3})$. (3 marks)

M1: Multiply numerator and denominator by $\sqrt{3}$ [1]

A1: Numerator becomes $\sqrt{3}$, denominator becomes 3 [1]

B1: Final answer: $\sqrt{3}/3$ [1]

6. The third term of a geometric progression is 24 and the fifth term is 96. Find the common ratio (assume positive). (4 marks)

M1: Let first term a , common ratio r . Third term: $ar^2 = 24$ [1]

M1: Fifth term: $ar^4 = 96$ [1]

M1: Divide: $(ar^4)/(ar^2) = r^2 = 96/24 = 4$ [1]

A1: $r = 2$ (positive) [1]

7. Solve the quadratic equation $x^2 - 4x - 5 = 0$ by completing the square. (4 marks)

M1: Write as $x^2 - 4x = 5$ [1]

M1: Add $(4/2)^2 = 4$ to both sides: $x^2 - 4x + 4 = 9$ [1]

M1: Factor: $(x-2)^2 = 9$ [1]

A1: $x - 2 = \pm 3 \rightarrow x = 5$ or $x = -1$ [1]

Alternative method/answer:

Alternative: Using quadratic formula: $x = [4 \pm \sqrt{(16+20)}]/2 = (4 \pm 6)/2 \rightarrow x=5$ or -1 (M1 for formula, A1 for roots)

8. Find the determinant of matrix $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$. (2 marks)

M1: Formula: $\det = (2)(5) - (3)(4)$ [1]

A1: $\det = 10 - 12 = -2$ [1]

9. Use the binomial expansion to find the coefficient of x^3 in the expansion of $(2 - x)^6$. (3 marks)

M1: Use general term: $T_{r+1} = C(6,r)(2)^{6-r}(-x)^r$ [1]

M1: For x^3 , $r=3$: $C(6,3)=20$, $(2)^3=8$, $(-x)^3 = -x^3$ [1]

A1: Coefficient = $20 * 8 * (-1) = -160$ [1]

10. y varies directly as the square of x and inversely as z . When $x=2$ and $z=4$, $y=3$. (4 marks)

Find y when $x=4$ and $z=8$.

M1: Setup: $y = k x^2 / z$ [1]

M1: Substitute given: $3 = k(4)/4 \rightarrow k=3$ [1]

M1: New values: $y = 3(16)/8$ [1]

A1: $y = 48/8 = 6$ [1]

11. Without using a calculator, find the value of $\sin 150^\circ + \cos 240^\circ$. (3 marks)

B1: $\sin 150^\circ = \sin(180-30) = \sin 30^\circ = 1/2$ [1]

B1: $\cos 240^\circ = \cos(180+60) = -\cos 60^\circ = -1/2$ [1]

A1: Sum = $1/2 + (-1/2) = 0$ [1]

12. The locus of a point P is given by $PA = PB$ where $A(1,2)$ and $B(3,4)$. The locus of P also satisfies the condition that P lies on the line $x = 2$. Find the coordinates of the points of intersection of these two loci. (3 marks)

M1: $PA=PB \rightarrow$ perpendicular bisector of AB . Midpoint $M(2,3)$ [1]

M1: Equation of bisector: slope = -1 , $y-3 = -(x-2) \rightarrow y = -x+5$ [1]

A1: Intersection with $x=2$: $y = -2+5=3$, so point $(2,3)$ [1]

13. A transformation is represented by the matrix $M = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$. Find the image of the point $P(2,-1)$ under this transformation. (2 marks)

M1: Multiply matrix by column vector: $M * (2, -1)^T$ [1]

A1: Image = $((2*2+1*(-1)), (3*2+4*(-1))) = (3,2)$ [1]

14. Given vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, find: (4 marks)

M1: (a) Find $2\mathbf{b} = (4, -2, 2)$ [1]

M1: $\mathbf{a} + 2\mathbf{b} = (5, 0, 1)$ [1]

A1: Magnitude = $\sqrt{(25+0+1)} = \sqrt{26}$ (accept ' $\sqrt{26}$ ') [1]

M1: (b) $\mathbf{a} - \mathbf{b} = (-1, 3, -4)$, magnitude = $\sqrt{26}$ [1]

A1: Unit vector = $(1/\sqrt{26})(-1, 3, -4)$ [1]

Alternative method/answer:

Note: award marks as 2 for (a) and 2 for (b). Total 4 marks.

15. In the diagram, AC is a diameter of the circle with centre O . angle $ABC = 70^\circ$. Find angle ACB . (2 marks)

B1: Angle in semicircle: $\angle ABC = 90^\circ$ (corrected from original problem, using $\angle BAC=70^\circ$). [1]

A1: $\angle ACB = 180^\circ - 90^\circ - 70^\circ = 20^\circ$ [1]

Alternative method/answer:

Alternative: If original $\angle ABC = 70^\circ$ and AC is diameter, $\angle ABC$ should be 90° , leading to inconsistency. Marker to accept any valid reasoning. Suggested correction: $\angle BAC = 70^\circ$.

- 16.** Find the scalar product of vectors $\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$. (2 marks)

M1: Dot product formula: $\mathbf{u} \cdot \mathbf{v} = 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6$ [1]

A1: $= 4 + 10 + 18 = 32$ [1]

SECTION II (50 marks)

- 17.** The position vectors of points A, B and C are $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\mathbf{c} = 2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$ respectively. Point D divides AB in the ratio 2:3 internally. Point E divides AC in the ratio 3:1 externally. (10 marks)

M1: Internal division: $\mathbf{d} = (3\mathbf{a} + 2\mathbf{b})/5$ [1]

A1: $\mathbf{d} = (11\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})/5$ [1]

M1: External division: $\mathbf{e} = (3\mathbf{c} - \mathbf{a})/2$ [1]

A1: $\mathbf{e} = (3\mathbf{i} + 13\mathbf{j} + 10\mathbf{k})/2$ [1]

M1: $\mathbf{DE} = \mathbf{e} - \mathbf{d}$ [1]

A1: $\mathbf{DE} = (-7/10)\mathbf{i} + (57/10)\mathbf{j} + 4\mathbf{k}$ [1]

M1: Magnitude $|\mathbf{DE}| = \sqrt{(-0.7)^2 + (5.7)^2 + 4^2}$ [1]

M1: Correct squares: $0.49 + 32.49 + 16 = 48.98$ [1]

A1: $\sqrt{48.98} \approx 7.00$ (2 sf or 3 sf correct) [1]

B1: Correct final answer with units [1]

Alternative method/answer:

Alternative: Using formula directly. Accept exact surd $\sqrt{(4898/100)} = \sqrt{4898}/10 = \sqrt{(2 \cdot 31 \cdot 79)}/10$, etc.

- 18.** A triangle with vertices P(1,2), Q(3,1) and R(2,4) undergoes two successive transformations. First, a reflection in the line $y = x$ represented by matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, followed by a rotation through 90° anti-clockwise about the origin represented by matrix $\mathbf{B} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. (10 marks)

M1: Combined matrix $= \mathbf{BA} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ [1]

A1: $= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ [1]

M1: Apply to P: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ [1]

A1: P'(-1,2) [1]

M1: Apply to Q: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ [1]

A1: Q'(-3,1) [1]

M1: Apply to R: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ [1]

A1: R'(-2,4) [1]

M1: Part (c): Let S(x,y). Then S' = (-x, y) = (3,1) [1]

A1: x = -3, y = 1, so S(-3,1) [1]

- 19.** In the diagram below, O is the centre of the circle. AB is a chord of length 12 cm. OC is perpendicular to AB and meets AB at C. Given that OC = 4 cm, (10 marks)

M1: $AC = AB/2 = 6$ cm [1]

M1: Right triangle OCA: $OA^2 = OC^2 + AC^2$ [1]

A1: $OA = \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13}$ cm [1]

M1: Power of point D: $DA^2 = DB \cdot (DB + AB)$ [1]

M1: Substitute: $DA^2 = 6 \cdot (6 + 12) = 108$ [1]

A1: $DA = \sqrt{108} = 6\sqrt{3}$ cm [1]

B1: Correct units (cm) [1]

B1: Final answers in surd form or decimal equivalent [1]

Alternative method/answer:

Alternative: Using tangent-secant theorem: $AD^2 = DC \cdot DB$ where $DC = AB + BD = 18$, $DB = 6 \rightarrow AD^2 = 108$.

Accept decimal approximations (10.39 cm) if consistent.

20. (10 marks)

M1: (a) Binomial expansion: $(1+3x)^5 = \sum_{r=0}^5 C(5,r)(1)^{5-r}(3x)^r$ [1]

M1: Up to x^3 : $1 + 5(3x) + 10(3x)^2 + 10(3x)^3$ [1]

A1: $= 1 + 15x + 90x^2 + 270x^3$ [1]

M1: (b) Set $1+3x = 1.03 \rightarrow x = 0.01$ [1]

M1: Substitute: $1 + 15(0.01) + 90(0.01)^2 + 270(0.01)^3$ [1]

M1: Correct computation: $1 + 0.15 + 0.009 + 0.00027$ [1]

A1: Sum = 1.15927 [1]

B1: Rounded to 4 dp: 1.1593 [1]

Alternative method/answer:

Alternative: Direct calculator (no marks for expansion part). For part (b), accept any consistent method if expansion is correct.

21. The population of a town is increasing at a rate of 8% per annum. The current population is 250,000. (10 marks)

M1: (a) Formula: $P = 250000(1.08)^n$ [1]

M1: $n=3$: $P = 250000(1.08)^3$ [1]

A1: $= 250000 \cdot 1.259712 = 314928$ (accept 314900 or 314928) [1]

M1: (b) Inequality: $250000(1.08)^n > 400000$ [1]

M1: $(1.08)^n > 1.6$ [1]

M1: Take logs: $n \log 1.08 > \log 1.6$ [1]

A1: $n > \log 1.6 / \log 1.08 \approx 0.20412 / 0.03342 \approx 6.11$ [1]

B1: Since n is integer, $n = 7$ years [1]

Alternative method/answer:

Alternative: Using trial and error: $(1.08)^6 = 1.5869$, $(1.08)^7 = 1.7138 > 1.6 \rightarrow n = 7$ (M1 for each correct power, A1 for conclusion).

22. A sum of money invested at compound interest amounts to $\pounds 13,310$ after 3 years and $\pounds 14,641$ after 4 years. (10 marks)

M1: (a) Let r be rate. $A_2/A_1 = (1+r) = 14641/13310$ [1]

A1: $= 1.1 \rightarrow r = 0.1 = 10\%$ [1]

M1: (b) $P(1.1)^3 = 13310$ [1]

A1: $P = 13310/1.331 = 10000$ [1]

M1: (c) Amount after 6 years: $A = 10000(1.1)^6$ [1]

M1: $= 10000 \cdot 1.771561$ [1]

A1: $= 17715.61$ (accept 17716) [1]

Alternative method/answer:

Alternative: Using formula directly. Accept decimal rounding as per question.

23. (10 marks)

M1: Let $P(x,y)$. Distance to A: $\sqrt{(x-2)^2 + (y-3)^2}$ [1]

M1: Distance to line $x=5$: $|x-5|$ [1]

M1: Set equal: $\sqrt{(x-2)^2 + (y-3)^2} = |x-5|$ [1]

M1: Square: $(x-2)^2 + (y-3)^2 = (x-5)^2$ [1]

M1: Expand: $x^2 - 4x + 4 + y^2 - 6y + 9 = x^2 - 10x + 25$ [1]

A1: Simplify: $y^2 - 6y + 6x - 12 = 0$ or equivalent [1]

M1: Complete square: $(y-3)^2 - 9 + 6x = 12 \rightarrow (y-3)^2 = 21 - 6x$ [1]

A1: Final equation: $(y-3)^2 = 21 - 6x$ [1]

B1: Description: parabola opening left, vertex at (3.5,3) [1]

B1: Sketch with correct orientation, vertex, and axis of symmetry [1]

Alternative method/answer:

Alternative: Rearranged as $x = -(1/6)(y-3)^2 + 3.5$. Accept any equivalent form.

24. A quadratic equation $x^2 + kx + 9 = 0$ has two equal roots.

(10 marks)

M1: (a) Equal roots $\rightarrow$ discriminant = 0: $k^2 - 36 = 0$ [1]

A1: $k = \pm 6$ [1]

M1: (b) For $k=6$: expression = $x^2 + 6x + 9 = (x+3)^2$ [1]

A1: Root: $x = -3$ (repeated) [1]

M1: For $k=-6$: expression = $x^2 - 6x + 9 = (x-3)^2$ [1]

A1: Root: $x = 3$ (repeated) [1]

M1: (c) Perfect square $\Rightarrow$ discriminant=0 $\Rightarrow k=\pm 6$ [1]

A1: Factorised as $(x+3)^2$ for $k=6$, $(x-3)^2$ for $k=-6$ [1]