

EDUZEDCO

MATHEMATICAL ACTIVITIES

Form 4 End Term Examination

Time: 2 ½ Hours

Name: Adm No:
 School: Class:
 Signature: Date:

INSTRUCTIONS TO CANDIDATES

- (a) Write your name and index number in the spaces provided at the top of this page.
- (b) Write your school name, sign and write the date of the examination in the spaces provided above.
- (c) This paper consists of **Two** sections: **Section I** and **Section II**.
- (d) Answer **ALL** questions in Section I and any **five questions** from Section II.
- (e) Marks may be given for correct working, relevant points, and clear presentation.
- (h) This paper consists of **4 printed pages**.
- (i) Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.
- (j) Candidates should answer the questions in English.

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

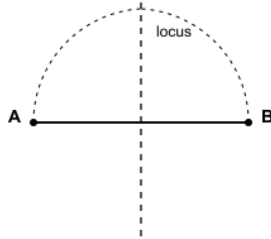
SECTION II

17	18	19	20	21	22	23	24	Total	Grand Total

SECTION II

Answer any five questions from this section.

17. The displacement s metres of a particle moving along a straight line after t seconds is given by $s = t^3 - 6t^2 + 9t + 2$, where $t \geq 0$. (a) Find the velocity of the particle after 2 seconds. (b) (c) Find the acceleration of the particle when $t = 3$. (d) Calculate the distance travelled by the particle during the first 3 seconds. [10 marks]



Find the velocity of the particle after 2 seconds.

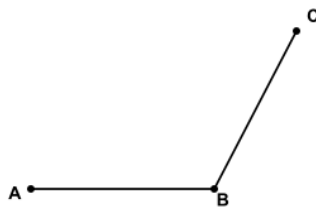
Determine the time(s) when the particle is at rest.

Find the acceleration of the particle when $t = 3$.

Calculate the distance travelled by the particle during the first 3 seconds.

18. Points A , B and C have position vectors $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} 7 \\ -4 \\ 6 \end{pmatrix}$ respectively.

- (a) Find the vector $\vec{AB}$. (b) Find the coordinates of point P which divides AB externally in the ratio $2 : 1$. (c) Show that points A , B and C are collinear. [10 marks]

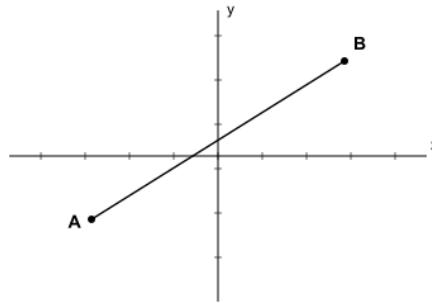


Find the vector $\vec{AB}$.

Find the coordinates of point P which divides AB externally in the ratio $2:1$.

Show that points A , B and C are collinear.

19. The graph of $y = 3\sin(2x - 60^\circ) + 1$ for $0^\circ \leq x \leq 360^\circ$ is drawn. (a) (b) (c) Use your graph to solve the equation $3\sin(2x - 60^\circ) + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$. [10 marks]



State the amplitude, period and phase shift of the wave.

Sketch the graph for the given range. Show clearly the maximum and minimum points, and intercepts with the axes.

Use your graph to solve the equation $3\sin(2x - 60^\circ) + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$.

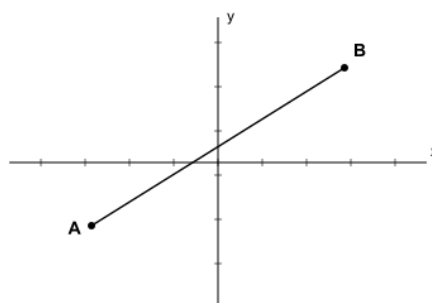
20. A population of bacteria initially numbers 5000 and increases at a rate proportional to the number present. After 3 hours, the population is 8000. (a) Find the growth constant k . (b) (c) [10 marks]

Find the growth constant k .

How many bacteria will there be after 5 hours?

How long will it take for the population to double?

21. The curve $y = x^2 - 4x + 5$ meets the x -axis at points where $x = 1$ and $x = 4$. (a) Find the area enclosed by the curve and the x -axis between $x = 1$ and $x = 4$. (b) Find the volume generated when the region in (a) is rotated completely about the x -axis. [10 marks]



Find the area enclosed by the curve and the x -axis between $x=1$ and $x=4$.

Find the volume generated when the region in (a) is rotated completely about the x -axis.

22. In a certain year, the income tax rates for residents in Kenya were as follows: A worker earns a gross monthly salary of Ksh 65,000. He is entitled to a personal relief of Ksh 1,280 per month. He also contributes Ksh 400 to NHIF and Ksh 600 to NSSF (both deducted before tax). (a) (b) (c) [10 marks]

Monthly taxable income (Ksh)	Tax rate (%)
0 - 12,298	10
12,299 - 23,885	15
23,886 - 35,472	20
35,473 - 47,059	25
Over 47,059	30

Calculate the taxable income.

Calculate the total income tax (PAYE) payable.

Determine the net salary after all deductions.

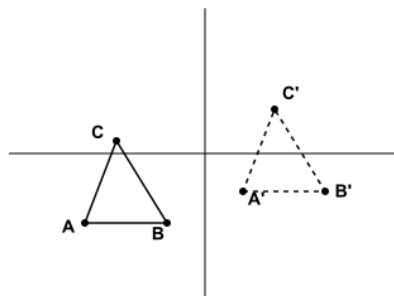
23. (a) Expand $(1 - 2x)^5$ in ascending powers of x up to the term in x^5 . (b) Use your expansion to evaluate $(0.98)^5$ correct to 4 decimal places. (c) Find the value of x for which $(1 - 2x)^5 = 0.5$ correct to 2 decimal places using the expansion up to x^2 . [10 marks]

Expand $(1 - 2x)^5$ in ascending powers of x up to the term in x^5 .

Use your expansion to evaluate $(0.98)^5$ correct to 4 decimal places.

Find the value of x for which $(1 - 2x)^5 = 0.5$ correct to 2 decimal places using the expansion up to x^2 .

24. A triangle with vertices $P(1, 0)$, $Q(2, 1)$ and $R(0, 2)$ is mapped onto triangle $P'Q'R'$ by transformation $T = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ followed by $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. (a) (b) Determine the coordinates of P' , Q' and R' . (c) [10 marks]



Find the matrix representing the combined transformation.

Determine the coordinates of P' , Q' and R' .

Describe fully the single transformation represented by the combined matrix.

None MATHEMATICAL ACTIVITIES MARKING SCHEME

Form 4 Paper 2

SECTION I (50 marks)

1. (3 marks)

Multiply both sides by c: $cy = ax + b$ (M1) [1]

Subtract b: $ax = cy - b$ (M1) [1]

Divide by a: $x = (cy - b)/a$ (A1) [1]

Alternative method/answer:

Accept correct manipulation leading to $x = (cy - b)/a$

2. (3 marks)

Combine logs: $\log_2 (x(x-2)) = 3$ (M1) [1]

Exponential form: $x(x-2) = 2^3 = 8$ (M1) [1]

Solve quadratic: $x^2 - 2x - 8 = 0 \rightarrow x = 4$ or $x = -2$; reject $x = -2$; answer $x = 4$ (A1) [1]

Alternative method/answer:

Accept correct solution using log properties

3. (2 marks)

Write as 5.6×10^{-5} (M1 for coefficient) [1]

Correct exponent -5 (A1) [1]

Alternative method/answer:

Accept 5.6e-5

4. (4 marks)

Express 27 as 3^x : $3^x = 3^{3(2x-1)}$ (M1) [1]

Equate exponents: $x = 6x - 3$ (M1) [1]

Solve linear equation: $-5x = -3$ (M1) [1]

Correct solution $x = 3/5$ (A1) [1]

Alternative method/answer:

Accept taking logs both sides

5. (3 marks)

Form equation: $y = kx^2$ (M1) [1]

Substitute $y=18$, $x=3$: $18 = k \cdot 9 \rightarrow k=2$ (M1) [1]

Find y when $x=4$: $y = 2 \cdot (16) = 32$ (A1) [1]

Alternative method/answer:

Accept direct proportion reasoning

6. (3 marks)

Reference angle = $180^\circ - 150^\circ = 30^\circ$ (M1) [1]

Correct quadrant: sin positive in 2nd quadrant (M1) [1]

$\sin 150^\circ = \sin 30^\circ = 1/2$ (A1) [1]

Alternative method/answer:

Accept using unit circle or symmetry

7. (3 marks)

Identify $a = 2$, $r = 3$ (M1) [1]

Use formula $S_n = a(r^n - 1)/(r - 1)$ (M1) [1]

$S_5 = 2(243 - 1)/2 = 242$ (A1) [1]

Alternative method/answer:

Accept summing terms directly if all correct

8. (4 marks)

Determinant formula: $\det = ad - bc$ (M1) [1]

Compute: $2 \times 4 - 3 \times 1 = 8 - 3 = 5$ (A1) [1]

Inverse formula: $A^{-1} = \frac{1}{\det} \begin{bmatrix} d, -b \\ -c, a \end{bmatrix}$ (M1) [1]

Write $A^{-1} = \frac{1}{5} \begin{bmatrix} 4, -3 \\ -1, 2 \end{bmatrix}$ (A1) [1]

Alternative method/answer:

Accept correct inverse with elements simplified

SECTION II (50 marks)

17. The displacement s metres of a particle moving along a straight line after t seconds is given by $s = t^3 - 6t^2 + 9t + 2$, where $t \geq 0$. (a) Find the velocity of the particle after 2 seconds. (b) (c) Find the acceleration of the particle when $t = 3$. (d) Calculate the distance travelled by the particle during the first 3 seconds. (10 marks)

Differentiate to get $v = 3t^2 - 12t + 9$ (M1) [1]

Substitute $t=2$: $v = 3(4) - 24 + 9 = -3$ m/s (A1) [1]

Set $v=0$: $3t^2 - 12t + 9 = 0 \rightarrow t^2 - 4t + 3 = 0$ (M1) [1]

Solve: $(t-1)(t-3)=0 \rightarrow t=1, t=3$ (A1) [1]

Differentiate v to get $a = 6t - 12$ (M1) [1]

Substitute $t=3$: $a = 6(3) - 12 = 6$ m/s² (A1) [1]

Determine sign of v for $0 \leq t < 1$: positive; $1 < t < 3$: negative (M1) [1]

Set up distance $= \int_0^1 v \, dt + \int_1^3 (-v) \, dt$ (M1) [1]

Integrate: $\int v \, dt = t^3 - 6t^2 + 9t$; evaluate $\int_0^1 = 4, \int_1^3 = -4$ (M1) [1]

Total distance $= 4 + 4 = 8$ m (A1) [1]

Alternative method/answer:

Alternative for (d): find displacement at $t=0, 1, 3$ and sum absolute differences; full marks if correct

18. Points A, B and C have position vectors $\text{veca} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, $\text{vecb} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$ and $\text{vecc} = \begin{pmatrix} 7 \\ -4 \\ 6 \end{pmatrix}$ respectively. (a) Find the vector vecAB . (b) Find the coordinates of point P which divides AB externally in the ratio 2:1. (c) Show that points A, B and C are collinear. (10 marks)

$AB = b - a = (4-1, -1-2, 3-0) = (3, -3, 3)$ (M1 for subtraction, A1 for correct) [2]

External division formula: $P = \frac{2b - 1a}{2-1} = 2b - a$ (M1) [1]

Compute $P = (8-1, -2-2, 6-0) = (7, -4, 6)$ (M1, A1) [2]

Find $AB = (3, -3, 3)$ and $BC = c - b = (3, -3, 3)$ (M1) [1]

Show AB is parallel to BC (common direction) (M1) [1]

Show they share point B (common point) (M1) [1]

Therefore A, B, C are collinear (A1) [1]

Alternative method/answer:

Alternative for (c): find ratio AB:BC = 1:1 and show B is common; or use scalar triple product = 0

19. The graph of $y = 3\sin(2x - 60^\circ) + 1$ for $0^\circ \leq x \leq 360^\circ$ is drawn. (a) (b) (c) Use your graph to solve the equation $3\sin(2x - 60^\circ) + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$. (10 marks)

State amplitude = 3 (B1) [1]

State period = $360/2 = 180^\circ$ (B1) [1]

State phase shift = 60° to the right (B1) [1]

Draw axes with correct scales (M1) [1]

Correct shape (sine wave) with 2 cycles in 360° (M1) [1]

Show maximum points at $y=4$, $x=75^\circ$, 255° (M1) [1]

Show minimum points at $y=-2$, $x=165^\circ$, 345° (M1) [1]

Read solutions from graph where $y=0$: $x \approx 110^\circ$, 260° (M1 for reading, A1 for both values) [2]

Alternative method/answer:

Accept exact solutions using inverse sine: $x = 1/2(\arcsin(-1/3) + 60^\circ)$ etc.

- 20.** A population of bacteria initially numbers 5000 and increases at a rate proportional to the number present. After 3 hours, the population is 8000. (a) Find the growth constant k . (b) (c) (10 marks)

Write exponential growth model $N = N_0 e^{kt}$ (M1) [1]

Substitute $N_0=5000$, $N=8000$, $t=3$: $8000 = 5000 e^{3k}$ (M1) [1]

Solve for k : $e^{3k}=1.6 \rightarrow 3k = \ln 1.6 \rightarrow k = (\ln 1.6)/3$ (M1) [1]

Compute $k \approx 0.1567$ (A1) [1]

Substitute $t=5$: $N = 5000 e^{5k}$ (M1) [1]

Compute $N \approx 5000 * e^{0.7835} \approx 5000 * 2.189 \approx 10945$ (A1) [1]

Set up doubling: $10000 = 5000 e^{kt} \rightarrow e^{kt}=2$ (M1) [1]

Solve: $kt = \ln 2 \rightarrow t = \ln 2 / k$ (M1) [1]

Compute $t \approx 0.6931/0.1567 \approx 4.42$ hours (A1) [1]

Alternative method/answer:

Accept use of growth constant in different form, e.g., using doubling time formula

- 21.** The curve $y = x^2 - 4x + 5$ meets the x -axis at points where $x=1$ and $x=4$. (a) Find the area enclosed by the curve and the x -axis between $x=1$ and $x=4$. (b) Find the volume generated when the region in (a) is rotated completely about the x -axis. (10 marks)

Area = $\int_1^4 (x^2 - 4x + 5) dx$ (M1) [1]

Integrate: $[x^3/3 - 2x^2 + 5x]$ (M1) [1]

Substitute upper limit (4): $64/3 - 32 + 20 = 64/3 - 12$ (M1) [1]

Substitute lower limit (1): $1/3 - 2 + 5 = 1/3 + 3$ (M1) [1]

Subtract: $(64/3 - 12) - (1/3 + 3) = 63/3 - 15 = 21 - 15 = 6$ (A1) [1]

Volume = $\pi \int_1^4 (x^2 - 4x + 5)^2 dx$ (M1) [1]

Expand: $x^4 - 8x^3 + 26x^2 - 40x + 25$ (M1) [1]

Integrate: $[x^5/5 - 2x^4 + 26x^3/3 - 20x^2 + 25x]$ (M1) [1]

Evaluate at 4 and 1 and subtract: difference = 15.6 (M1) [1]

Volume = 15.6π cubic units (A1) [1]

Alternative method/answer:

Accept exact fraction $78\pi/5$ if simplified

- 22.** In a certain year, the income tax rates for residents in Kenya were as follows: A worker earns a gross monthly salary of Ksh 65,000. He is entitled to a personal relief of Ksh 1,280 per month. He also contributes Ksh 400 to NHIF and Ksh 600 to NSSF (both deducted before tax). (a) (b) (c) (10 marks)

Taxable income = gross - NHIF - NSSF = $65,000 - 400 - 600 = 64,000$ (M1, A1) [2]

First band: $12,298 \times 10\% = 1,229.80$ (M1) [1]

Second band: $11,587 \times 15\% = 1,738.05$ (M1) [1]

Third band: $11,587 \times 20\% = 2,317.40$ (M1) [1]

Fourth band: $11,587 \times 25\% = 2,896.75$ (M1) [1]

Remaining: $64,000 -$