

EDUZEDCO
121/2 MATHEMATICS
Paper 2
Form 4 End Term Examination

Time: 2 ½ Hours

Name: Adm No:
 School: Class:
 Signature: Date:

INSTRUCTIONS TO CANDIDATES

- (a) Write your name and index number in the spaces provided at the top of this page.
- (b) Write your school name, sign and write the date of the examination in the spaces provided above.
- (c) This paper consists of **Two** sections: **Section I** and **Section II**.
- (d) Answer **ALL** questions in Section I and any **five questions** from Section II.
- (e) Marks may be given for correct working, relevant points, and clear presentation.
- (f) Non-programmable silent electronic calculators may be used where necessary.
- (g) KNEC Mathematical Tables may be used.
- (h) This paper consists of **8 printed pages**.
- (i) Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.
- (j) Candidates should answer the questions in English.

For Examiner's Use Only

SECTION I

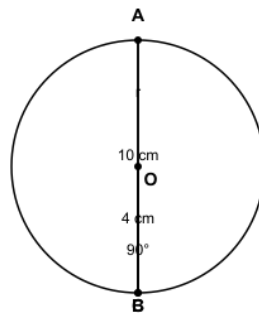
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

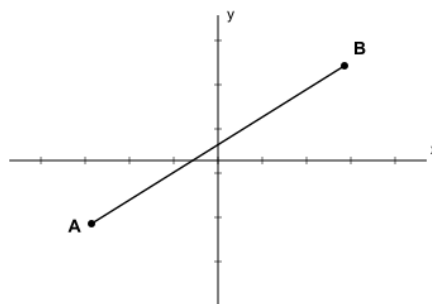
17	18	19	20	21	22	23	24	Total	Grand Total

SECTION I

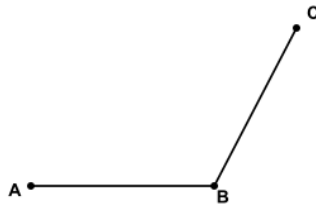
1. Make y the subject of the formula: $x = \frac{3y-2}{y+1}$.
2. Simplify $\frac{3}{\sqrt{5}-2}$ by rationalising the denominator.
3. A rectangular field measures 45.0 m by 32.0 m, each measured to the nearest metre. Calculate the percentage error in its area.
4. y varies partly directly as x and partly inversely as x . When $x = 1$, $y = 7$; when $x = 2$, $y = 5$. Find the equation connecting x and y .
5. The third term of a geometric progression is 12 and the sixth term is 96. Find the first term and the common ratio.
6. Solve the equation $2x^2 - 5x - 3 = 0$ by completing the square.
7. Expand $(2 - x)^5$ in ascending powers of x up to the term in x^3 .
8. In the diagram below, O is the centre of the circle. AB is a chord of length 10 cm, and OC is perpendicular to AB meeting AB at C . Given that $OC = 4$ cm, find the radius of the circle.



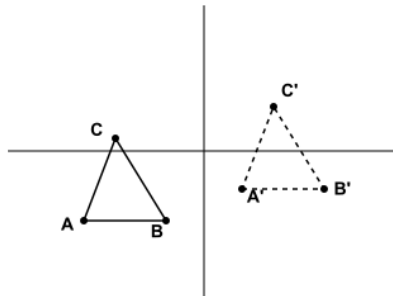
9. Solve the equation $\sin(2\theta) = 0.5$ for $0^\circ \leq \theta \leq 360^\circ$.



10. Given vectors $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$, find $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$.



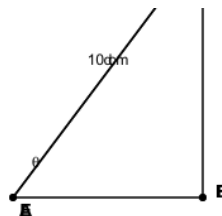
11. A transformation is represented by matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. Find the image of the point $(3, -4)$ under this transformation.



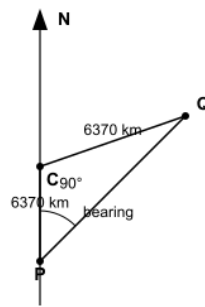
12. A bag contains 3 red balls, 5 blue balls, and 2 green balls. Two balls are drawn without replacement. Find the probability that the second ball is red given that the first ball is blue.

13. Evaluate $\int_0^1 (3x^2 + 2x) dx$.

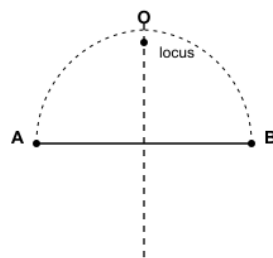
14. A rectangular box has dimensions 6 cm, 8 cm, and 10 cm. Find the angle between the diagonal of the box and the base plane.



15. Two cities P and Q lie on the equator. P is at longitude 30°E , Q at longitude 60°W . Find the distance between them along the equator, taking the Earth's radius as 6370 km.



- 16.** Construct on paper the locus of points equidistant from two fixed points A and B and also equidistant from two lines l_1 and l_2 that intersect at O . Describe the final locus.



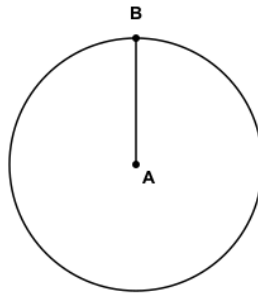
SECTION II

Answer any five questions from this section.

17. In a certain country, income tax is charged on annual income as follows:

Annual Income (Ksh)	Tax Rate (%)
First 120,000	10%
Next 100,000	15%
Next 80,000	20%
Above 300,000	25%

A person earns a monthly salary of Ksh 35,000 and is entitled to a personal relief of Ksh 1,200 per month. (a) (2 marks) (b) (4 marks) (c) (2 marks) (d) (2 marks) [10 marks]



Calculate the annual taxable income.

Calculate the total tax payable per annum before relief.

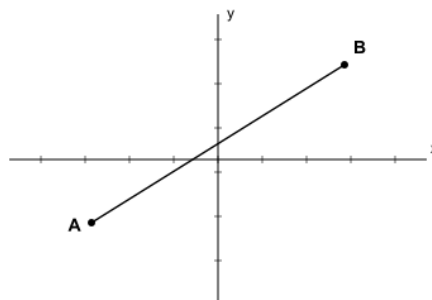
Determine the net tax payable per annum after relief.

Hence, find the net monthly tax payable.

18. The heights (in cm) of 50 students were recorded as follows:

Height (cm)	Frequency
140-144	4
145-149	8
150-154	12
155-159	14
160-164	7
165-169	5

(a) (4 marks) (b) (2 marks) (c) (4 marks) [10 marks]

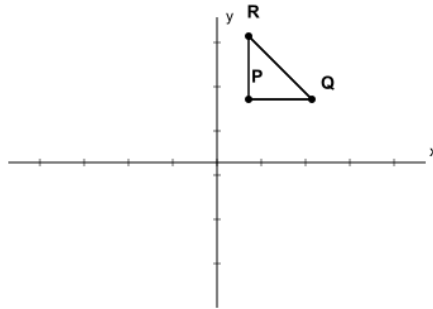


Complete the cumulative frequency table and draw the ogive.

Using the ogive, estimate the median height.

Calculate the variance and standard deviation of the heights.

19. Triangle PQR has vertices $P(1, 2)$, $Q(3, 2)$, $R(1, 4)$. It is transformed by matrix $M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ followed by matrix $N = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. (a) Determine the image of triangle PQR under the combined transformation NM . (4 marks) (b) (2 marks) (c) (4 marks) [10 marks]

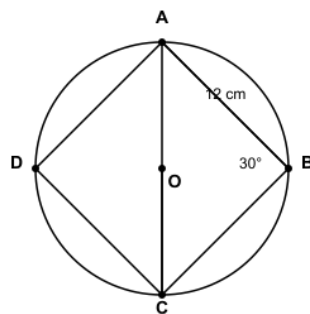


Determine the image of triangle PQR under the combined transformation NM .

Find the single matrix that represents the combined transformation.

Show that the area of the image is equal to the area of the original triangle.

20. In the diagram below, O is the centre of the circle. AB is a diameter. C is a point on the circle such that $\angle ABC = 30^\circ$. D is a point on the circle such that CD is parallel to AB . (a) Find the size of $\angle AOC$. (3 marks) (b) Find the size of $\angle ADC$. (3 marks) (c) If $AB = 12$ cm, find the length of CD . (4 marks) [10 marks]



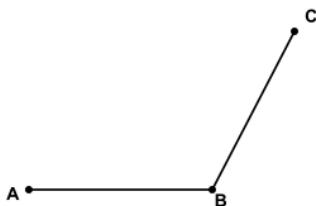
Find the size of angle AOC .

Find the size of angle ADC .

If $AB = 12$ cm, find the length of CD .

21. Points A , B , C have position vectors $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$. (a) Find the vector $\vec{AB}$ and $\vec{BC}$. (2 marks) (b) Show that A , B , C are collinear. (3 marks) (c) Find the ratio $AB : BC$. (3 marks)

(d) Find the position vector of the point P that divides AC externally in the ratio $2 : 1$. (2 marks) [10 marks]



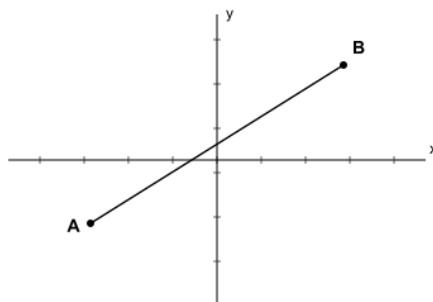
Find the vector AB and BC .

Show that A, B, C are collinear.

Find the ratio $AB:BC$.

Find the position vector of the point P that divides AC externally in the ratio $2:1$.

22. Consider the function $f(x) = 2\sin(3x) + 1$ for $0^\circ \leq x \leq 360^\circ$. (a) State the amplitude and period of $f(x)$. (2 marks) (b) Sketch the graph of $f(x)$ for the given interval. (4 marks) (c) Solve the equation $2\sin(3x) + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$. (4 marks) [10 marks]

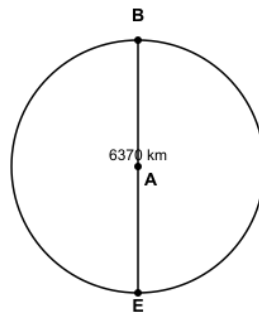


State the amplitude and period of $f(x)$.

Sketch the graph of $f(x)$ for the given interval.

Solve the equation $2\sin(3x) + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$.

23. Two towns A and B lie on the equator. A is at 30°E and B is at 45°E . Another town C is at 30°E , 60°N . (Take radius of earth = 6370 km, $\pi = 3.142$) (a) Find the distance between A and B along the equator. (3 marks) (b) Find the distance between A and C along the meridian. (3 marks) (c) Find the shortest distance between B and C along the great circle. (4 marks) [10 marks]

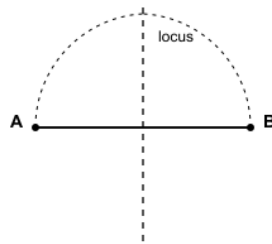


Find the distance between A and B along the equator.

Find the distance between A and C along the meridian.

Find the shortest distance between B and C along the great circle.

24. A particle moves along a straight line such that its velocity v m/s is given by $v = 3t^2 - 12t + 9$, where t is time in seconds ($t \geq 0$). (a) (1 mark) (b) (2 marks) (c) (3 marks) (d) (4 marks) [10 marks]



Find the initial velocity of the particle.

Find the times when the particle is momentarily at rest.

Determine the displacement of the particle during the first 4 seconds.

Calculate the total distance travelled during the first 4 seconds.

121/2 MATHEMATICS MARKING SCHEME

Form 4 Paper 2

SECTION I (50 marks)

1. Make y the subject of the formula: $x = (3y - 2)/(y + 1)$. (2 marks)

Multiply both sides by $(y+1)$: $x(y+1) = 3y - 2$ [M1]

Expand and collect y terms: $xy + x = 3y - 2 \rightarrow xy - 3y = -2 - x$ [M1]

Factor y : $y(x-3) = -(x+2) \rightarrow y = -(x+2)/(x-3) = (x+2)/(3-x)$ [A1]

Alternative method/answer:

$y = (x+2)/(3-x)$ or $y = -(x+2)/(x-3)$

2. Simplify $(3)/(\sqrt{5} - 2)$ by rationalising the denominator. (3 marks)

Multiply numerator and denominator by conjugate $(\sqrt{5}+2)$: $(3(\sqrt{5}+2))/((\sqrt{5}-2)(\sqrt{5}+2))$ [M1]

Denominator = $(\sqrt{5})^2 - 2^2 = 5 - 4 = 1$ [A1]

Result = $3(\sqrt{5}+2) = 3\sqrt{5} + 6$ [A1]

3. A rectangular field measures 45.0 m by 32.0 m, each measured to the nearest metre. (3 marks)

Calculate the percentage error in its area.

Maximum possible length = 45.5 m, minimum = 44.5 m; width max = 32.5 m, min = 31.5 m [B1]

Maximum area = $45.5 \times 32.5 = 1478.75 \text{ m}^2$; minimum area = $44.5 \times 31.5 = 1401.75 \text{ m}^2$ [M1]

Absolute error = $(1478.75 - 1401.75)/2 = 38.5 \text{ m}^2$; percentage error = $(38.5/1440) \times 100 = 2.674\% \approx 2.67\%$ [A1]

Alternative method/answer:

Accept 2.67% or 2.7%

4. y varies partly directly as x and partly inversely as x . When $x = 1$, $y = 7$; when $x = 2$, $y = 5$. Find the equation connecting x and y . (3 marks)

Let $y = ax + b/x$. Substitute: $7 = a + b$; $5 = 2a + b/2$ [M1]

Multiply second by 2: $10 = 4a + b$. Subtract first: $10 - 7 = 3a \rightarrow a = 1$ [M1]

Then $b = 6$. Equation: $y = x + 6/x$ [A1]

5. The third term of a geometric progression is 12 and the sixth term is 96. Find the first term and the common ratio. (3 marks)

Let first term = a , common ratio = r . Then $ar^2 = 12$, $ar^5 = 96$ [M1]

Divide: $(ar^5)/(ar^2) = r^3 = 96/12 = 8 \rightarrow r = 2$ [M1]

$a(2)^2 = 12 \rightarrow 4a = 12 \rightarrow a = 3$ [A1]

6. Solve the equation $2x^2 - 5x - 3 = 0$ by completing the square. (3 marks)

Divide by 2: $x^2 - (5/2)x - 3/2 = 0$. Add $(5/4)^2 = 25/16$ both sides: $x^2 - (5/2)x + 25/16 = 3/2 + 25/16 = 49/16$ [M1]

Left side: $(x - 5/4)^2 = 49/16$. Square root: $x - 5/4 = \pm 7/4$ [M1]

$x = 5/4 \pm 7/4 \rightarrow x = 3$ or $x = -1/2$ [A1]

7. Expand $(2 - x)^5$ in ascending powers of x up to the term in x^3 . (3 marks)

Use binomial: $(2 - x)^5 = \sum C(5,k) 2^{5-k} (-x)^k$. Terms: $k=0$: 32; $k=1$: $-80x$; $k=2$: $80x^2$; $k=3$: $-40x^3$ [M1]

Correct coefficients and signs for each term [M1]

Final expansion: $32 - 80x + 80x^2 - 40x^3$ [A1]

8. In the diagram below, O is the centre of the circle. AB is a chord of length 10 cm, and OC is perpendicular to AB meeting AB at C . Given that $OC = 4$ cm, find the radius of the circle. (4 marks)

C is midpoint of AB , so $AC = 5$ cm [B1]

In right triangle OCA, $OA^2 = OC^2 + AC^2 = 4^2 + 5^2 = 16 + 25 = 41$ [M1]

$OA = \sqrt{41}$ cm [A1]

Radius = $\sqrt{41}$ cm ≈ 6.403 cm [A1]

Alternative method/answer:

Accept $\sqrt{41}$ or 6.403 cm

- 9.** Solve the equation $\sin(2\theta) = 0.5$ for $0^\circ \leq \theta \leq 360^\circ$. (4 marks)

$\sin(2\theta) = 0.5 \rightarrow 2\theta = 30^\circ + 360^\circ k$ or $2\theta = 150^\circ + 360^\circ k$ [M1]

$\theta = 15^\circ + 180^\circ k$ or $\theta = 75^\circ + 180^\circ k$ [M1]

For $k=0$: $\theta = 15^\circ, 75^\circ$; for $k=1$: $\theta = 195^\circ, 255^\circ$ [M1]

All solutions in range: $15^\circ, 75^\circ, 195^\circ, 255^\circ$ [A1]

- 10.** Given vectors $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$, find $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$. (4 marks)

Dot product: $(2)(4) + (-1)(0) + (3)(-2) = 8 + 0 - 6 = 2$ [M1]

Cross product: $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 3 \\ 4 & 0 & -2 \end{vmatrix}$ [M1]

$\mathbf{i}((-1)(-2) - (3)(0)) - \mathbf{j}((2)(-2) - (3)(4)) + \mathbf{k}((2)(0) - (-1)(4)) = \mathbf{i}(2) - \mathbf{j}(-4-12) + \mathbf{k}(0+4) = (2, 16, 4)$ [M1]

Final: $\mathbf{a} \cdot \mathbf{b} = 2$, $\mathbf{a} \times \mathbf{b} = (2, 16, 4)$ [A1]

- 11.** A transformation is represented by matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. Find the image of the point $(3, -4)$ under this transformation. (4 marks)

Multiply M by column vector $(3, -4)$: $M \cdot \begin{bmatrix} 3 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \cdot 3 + 1 \cdot (-4) \\ 1 \cdot 3 + 2 \cdot (-4) \end{bmatrix}$ [M1]

Compute: $\begin{bmatrix} 6 - 4 \\ 3 - 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$ [M1]

Image point: $(2, -5)$ [A1]

- 12.** A bag contains 3 red balls, 5 blue balls, and 2 green balls. Two balls are drawn without replacement. Find the probability that the second ball is red given that the first ball is blue. (4 marks)

After drawing a blue ball, remaining: 3 red, 4 blue, 2 green = 9 balls [B1]

Probability second is red = $3/9 = 1/3$ [M1]

Final answer: $1/3$ [A1]

Alternative method/answer:

Accept 0.3333

- 13.** Evaluate $\int_0^1 (3x^2 + 2x) \, dx$. (4 marks)

Integrate: $\int (3x^2 + 2x) \, dx = x^3 + x^2 + C$ [M1]

Evaluate from 0 to 1: $(1^3 + 1^2) - (0 + 0) = 1 + 1 = 2$ [M1]

Final answer: 2 [A1]

- 14.** A rectangular box has dimensions 6 cm, 8 cm, and 10 cm. Find the angle between the diagonal of the box and the base plane. (4 marks)

Diagonal of base = $\sqrt{6^2 + 8^2} = 10$ cm [M1]

Space diagonal = $\sqrt{10^2 + 10^2} = 10\sqrt{2}$ cm [M1]

Angle between space diagonal and base: $\sin \theta = \text{opposite/hypotenuse} = 10/(10\sqrt{2}) = 1/\sqrt{2}$ [M1]

$\theta = 45^\circ$ [A1]

- 15.** Two cities P and Q lie on the equator. P is at longitude 30°E , Q at longitude 60°W . Find the distance between them along the equator, taking the Earth's radius as 6370 km. (4 marks)

Angular difference = $30^\circ - (-60^\circ) = 90^\circ$ [B1]

Arc length = $(\theta/360) \times 2\pi R = (90/360) \times 2\pi \times 6370 = (1/4) \times 2\pi \times 6370$ [M1]

Simplify: $(\pi \times 6370)/2$ km [M1]

Numerical value ≈ 10001.8 km [A1]

Alternative method/answer:

Accept $(\pi \times 6370)/2$ or 10001.8 km

- 16.** Construct on paper the locus of points equidistant from two fixed points A and B (4 marks)
and also equidistant from two lines l_1 and l_2 that intersect at O. Describe the final locus.

Locus of points equidistant from A and B is the perpendicular bisector of AB [B1]

Locus of points equidistant from lines l_1 and l_2 is the angle bisector of the angle between them [B1]

The intersection of these two loci gives the required points [M1]

Final locus: the intersection point(s) of the perpendicular bisector and the angle bisector [A1]

Alternative method/answer:

Accept description: the point(s) where the perpendicular bisector of AB meets the angle bisector of l_1 and l_2

SECTION II (50 marks)

- 17.** In a certain country, income tax is charged on annual income as follows: (10 marks)

begincenter
begin tabular{|||}\hline \textbf{Annual Income (Ksh)} & \textbf{Tax Rate (%)}\n\hline First 120,000 & 10%\nNext 100,000 & 15%\nNext 80,000 & 20%\nAbove 300,000 & 25%\n\hline endtabular
endcenter
A person earns a monthly salary of Ksh 35,000 and is entitled to a personal relief of Ksh 1,200 per month. (a) (2 marks) (b) (4 marks) (c) (2 marks) (d) (2 marks)

(a) Annual income = $35,000 \times 12 = 420,000$. Taxable income = 420,000 [B1]

(b) First 120,000: $120,000 \times 0.10 = 12,000$; Next 100,000: $100,000 \times 0.15 = 15,000$; Next 80,000: $80,000 \times 0.20 = 16,000$; Remaining 120,000: $120,000 \times 0.25 = 30,000$ [M1]

Total tax before relief = $12,000 + 15,000 + 16,000 + 30,000 = 73,000$ [A1]

(c) Annual relief = $1,200 \times 12 = 14,400$ [B1]

Net tax = $73,000 - 14,400 = 58,600$ [A1]

(d) Monthly net tax = $58,600 \div 12 = 4,883.33$ [A1]

Alternative method/answer:

Accept 4,883.33 or 4,883.3

- 18.** The heights (in cm) of 50 students were recorded as follows: (10 marks)

begin tabular{c|c}\hline Height (cm) & Frequency\n\hline 140-144 & 4\n145-149 & 8\n150-154 & 12\n155-159 & 14\n160-164 & 7\n165-169 & 5\n\hline endtabular
endcenter
(a) (4 marks) (b) (2 marks) (c) (4 marks)

(a) Cumulative frequencies: 4, 12, 24, 38, 45, 50. Plot points at upper boundaries: 144.5, 149.5, 154.5, 159.5, 164.5, 169.5 [M1]

Draw smooth ogive through points [M1]

(b) Median at cumulative frequency 25, read from ogive: approximately 155.5 cm [A1]

(c) Midpoints: 142, 147, 152, 157, 162, 167. Mean = $(142 \times 4 + 147 \times 8 + 152 \times 12 + 157 \times 14 + 162 \times 7 + 167 \times 5)/50 = 155.1$ [M1]

Variance = $\Sigma f(x-\mu)^2/n = [4(142-155.1)^2 + 8(147-155.1)^2 + 12(152-155.1)^2 + 14(157-155.1)^2 + 7(162-155.1)^2 + 5(167-155.1)^2]/50 = 48.99$ [M1]

Standard deviation = $\sqrt{48.99} = 7.00$ [A1]

Alternative method/answer:

Accept median 155-156 cm, variance 49.0, s.d. 7.0 cm

19. Triangle PQR has vertices P(1,2), Q(3,2), R(1,4). It is transformed by matrix $M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ followed by matrix $N = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. (a) Determine the image of triangle PQR under the combined transformation NM. (4 marks) (b) (2 marks) (c) (4 marks)

(a) $NM = N \times M = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$ [M1]

Image of P: $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; Q: $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$; R: $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ [M1]

(b) Single matrix = $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$ [A1]

(c) Area of original = $0.5 \times \text{base} \times \text{height} = 0.5 \times 2 \times 2 = 2$ [B1]

$\det(NM) = 2 \times 0 - (-1) \times 1 = 1$, area scale factor = $|\det| = 1$ [M1]

Area of image = $1 \times 2 = 2$ square units, same as original [A1]

Alternative method/answer:

Accept verification using coordinates: vectors $P'Q' = (4,2)$, $P'R' = (-2,0)$, area = $0.5|4 \times 0 - 2 \times (-2)| = 2$

20. In the diagram below, O is the centre of the circle. AB is a diameter. C is a point on the circle such that angle ABC = 30° . D is a point on the circle such that CD is parallel to AB. (a) Find the size of angle AOC. (3 marks) (b) Find the size of angle ADC. (3 marks) (c) If AB = 12 cm, find the length of CD. (4 marks)

(a) Angle ACB = 90° (angle in semicircle). In triangle ABC, angle CAB = $180 - 90 - 30 = 60^\circ$ [M1]

Angle AOC = $2 \times \text{angle ABC} = 60^\circ$ (angle at centre twice angle at circumference) [A1]

(b) CD $\parallel$ AB, so angle DCB = angle CBA = 30° (alternate angles). Angle ADC = angle ABC = 30° (angles subtended by same arc AC) [M1]

Angle ADC = 30° [A1]

(c) AB = 12 cm, radius = 6 cm. Triangle OCD is isosceles with OC = OD = 6 cm, angle COD = 60° [M1]

CD = 6 cm (equilateral triangle or using cosine rule: $CD^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \times \cos 60 = 36 + 36 - 72 \times 0.5 = 36$, so CD = 6) [A1]

Alternative method/answer:

Accept CD = 6 cm

21. Points A, B, C have position vectors $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$. (a) Find the vector $\vec{AB}$ and $\vec{BC}$. (2 marks) (b) Show that A, B, C are collinear. (3 marks) (c) Find the ratio AB:BC. (3 marks) (d) Find the position vector of the point P that divides AC externally in the ratio 2:1. (2 marks)

(a) $\vec{AB} = \vec{b} - \vec{a} = (3,3,3)$; $\vec{BC} = \vec{c} - \vec{b} = (3,3,3)$ [M1]

(b) $\vec{AB} = \vec{BC}$, so vectors are parallel and share point B, thus A, B, C are collinear [A1]

(c) Ratio AB:BC = 1:1 [A1]

(d) External division in ratio 2:1: $P = (2\vec{c} - \vec{a})/(2-1) = 2\vec{c} - \vec{a} = (14-1, 16-2, 18-3) = (13,14,15)$ [M1]

Position vector of P = (13,14,15) [A1]

22. Consider the function $f(x) = 2\sin(3x) + 1$ for $0^\circ \leq x \leq 360^\circ$. (a) State the amplitude and period of $f(x)$. (2 marks) (b) Sketch the graph of $f(x)$ for the given interval. (4 marks) (c) Solve the equation $2\sin(3x) + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$. (4 marks)

(a) Amplitude = 2, Period = $360/3 = 120^\circ$ [B1]

(b) Sketch: correct shape, key points at (0,1), (30,3), (90,-1), (120,1), etc. [M1]

Graph shows two full cycles in 0° to 360° [A1]

(c) Solve: $2\sin(3x) + 1 = 0 \rightarrow \sin(3x) = -1/2$ [M1]

$3x = 210^\circ + 360k$ or $330^\circ + 360k$ [M1]

$x = 70^\circ + 120k$ or $110^\circ + 120k$ [M1]

For $0 \leq x \leq 360$: $x = 70^\circ, 110^\circ, 190^\circ, 230^\circ, 310^\circ, 350^\circ$ [A1]

Alternative method/answer:

Accept solutions in degrees

23. Two towns A and B lie on the equator. A is at 30°E and B is at 45°E . (10 marks)

Another town C is at 30°E , 60°N . (Take radius of earth = 6370 km, $\pi = 3.142$)

(a) Find the distance between A and B along the equator. (3 marks) (b) Find the distance between A and C along the meridian. (3 marks) (c) Find the shortest distance between B and C along the great circle. (4 marks)

(a) Angular difference = 15° . Distance = $(15/360) \times 2\pi \times 6370 = (15/360) \times 2 \times 3.142 \times 6370 = 1668.5$ km [M1]

(b) Angular difference = 60° . Distance = $(60/360) \times 2\pi \times 6370 = (1/6) \times 2 \times 3.142 \times 6370 = 6674$ km [M1]

(c) Coordinates: B($0^\circ, 45^\circ\text{E}$), C($60^\circ\text{N}, 30^\circ\text{E}$). Use spherical cosine: $\cos(\text{arc}) = \sin 0 \sin 60 + \cos 0 \cos 60$
 $\cos(45-30) = 0 + 1 \times 0.5 \times \cos 15^\circ = 0.5 \times 0.9659 = 0.48295$ [M1]

Arc = $\arccos(0.48295) = 61.1^\circ$ [M1]

Distance = $(61.1/360) \times 2\pi \times 6370 = 6790$ km [A1]

Alternative method/answer:

Accept 1668.5 km, 6674 km, 6790 km

24. A particle moves along a straight line such that its velocity v m/s is given by $v =$ (10 marks)

$3t^2 - 12t + 9$, where t is time in seconds ($t \geq 0$). (a) (1 mark) (b) (2 marks) (c) (3 marks) (d) (4 marks)

(a) Initial velocity: $v(0) = 9$ m/s [B1]

(b) At rest: $v=0 \rightarrow 3t^2 - 12t + 9 = 0 \rightarrow t^2 - 4t + 3 = 0 \rightarrow (t-1)(t-3) = 0 \rightarrow t = 1, 3$ s [M1]

(c) Displacement = $\int v \, dt$ from 0 to 4 = $[t^3 - 6t^2 + 9t]$ from 0 to 4 = $(64 - 96 + 36) - 0 = 4$ m [M1]

(d) Distance: need absolute values. $\int |v| \, dt = \int_{\blacksquare}^1 v \, dt + \int_{\blacksquare}^3 -v \, dt + \int_{\blacksquare}^4 v \, dt$ [M1]

$\int_{\blacksquare}^1 v = [t^3 - 6t^2 + 9t]_{\blacksquare}^1 = 4$; $\int_{\blacksquare}^3 -v = [t^3 - 6t^2 + 9t]_{\blacksquare}^3 = (27 - 54 + 27) - (1 - 6 + 9) = 0 - 4 = -4$, absolute = 4; $\int_{\blacksquare}^4 v = [t^3 - 6t^2 + 9t]_{\blacksquare}^4 = (64 - 96 + 36) - (27 - 54 + 27) = 4 - 0 = 4$ [M1]

Total distance = $4 + 4 + 4 = 12$ m [A1]