

EDUZEDCO
121/2 MATHEMATICS
Paper 2
Form 4 End Term Examination

Time: 2 ½ Hours

Name: Adm No:
 School: Class:
 Signature: Date:

INSTRUCTIONS TO CANDIDATES

- (a) Write your name and index number in the spaces provided at the top of this page.
- (b) Write your school name, sign and write the date of the examination in the spaces provided above.
- (c) This paper consists of **Two** sections: **Section I** and **Section II**.
- (d) Answer **ALL** questions in Section I and any **five questions** from Section II.
- (e) Marks may be given for correct working, relevant points, and clear presentation.
- (f) Non-programmable silent electronic calculators may be used where necessary.
- (g) KNEC Mathematical Tables may be used.
- (h) This paper consists of **8 printed pages**.
- (i) Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.
- (j) Candidates should answer the questions in English.

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total	Grand Total

SECTION I

$$x = \frac{3y+2}{y-1}$$

1. Make y

the subject of the formula:

2. Simplify $\log_3 54 - \log_3 2$

3. A rectangle has sides 5.2 ± 0.1

3.8 ± 0.1

area.

cm and
cm. Calculate the maximum possible

4. y

x

z

$x = 3$

$z = 2$

$y = 6$

y

$x = 4$

$z = 5$

varies directly as
and inversely as

. When

and

,

. Find

when

and

.

5. The third term of a geometric progression is

12

48

and the fifth term is

. Find the common ratio.

6. Solve the equation $x^2 - 6x + 4 = 0$

square.

by completing the

7. Find the coefficient of x^3

of $(1 + 2x)^5$

in the expansion

.

8. In the diagram below, AB

circle with centre O

$AB = 10$

13

O

AB

is a chord of a

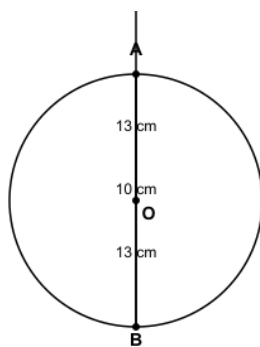
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cm and the radius is

cm. Find the distance from

to

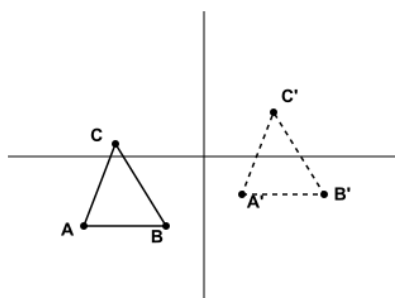
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9. Solve $\sin x = 0.5$ for $0^\circ \leq x \leq 360^\circ$.

10. Given $\vec{p} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$, find $\vec{p} + 2\vec{q}$.

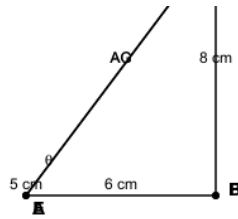
11. A transformation is represented by matrix $M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Find the image of the point $(2, 3)$ under M .



12. A bag contains 5 red and 3 blue marbles. Two marbles are drawn without replacement. Find the probability that both are red.

13. Evaluate $\int_1^2 (3x^2 + 2x) dx$.

14. A rectangular block has dimensions $AB = 6$ cm, $BC = 8$ cm, $AE = 5$ cm. Find the angle between diagonal AG and the base $ABCD$.



15. Two points on the equator have longitudes

30°

60°

the equator, taking the Earth's radius as 6370 km. (Use

$\pi = 3.142$

E and

E. Find the distance between them along

.)

16. Construct the locus of a point P

moves such that $PA = PB$

A

B

5

P

$PA = 3$

conditions?

that

where

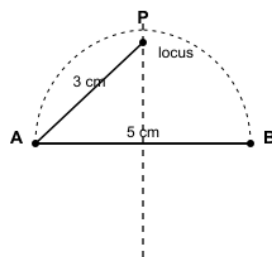
and

are fixed points

cm apart. Then construct the locus of

such that

cm. How many points satisfy both



SECTION II

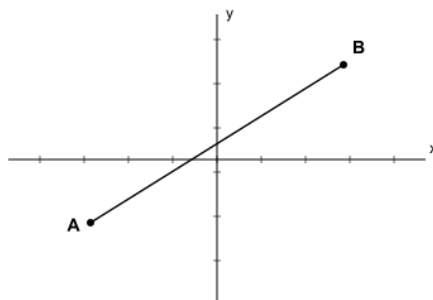
Answer any five questions from this section.

17. In a certain year, the income tax rates for a country were as follows: [10 marks]

Mr. Kamau earns a monthly basic salary of Ksh 45,000 and a house allowance of Ksh 15,000. He is entitled to a personal relief of Ksh 1,280 per month. Calculate his monthly PAYE.

In addition, Mr. Kamau contributes 5% of his basic salary to a registered pension scheme. Determine his net monthly income after all deductions.

18. The following table shows the distribution of marks scored by 50 students in a mathematics test: [10 marks]



Complete the cumulative frequency table and draw a cumulative frequency curve (ogive) for the data.

Using the ogive, estimate the median mark.

Calculate the interquartile range.

Determine the number of students who scored above 70 marks.

19. A triangle with vertices $A(1, 2)$

$B(3, 4)$

$C(5, 1)$

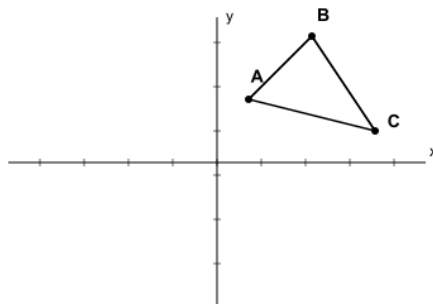
T_1

$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

, followed by T_2

$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

represented by matrix $\begin{pmatrix} 0 & 2 \\ 0 & 2 \end{pmatrix}$. [10 marks]



Determine the image of triangle ABC

T_1

after

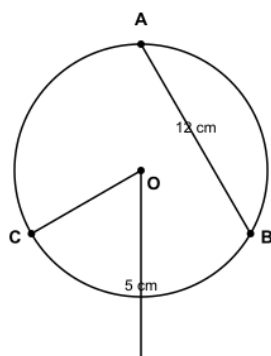
Determine the image after T_2 applied to the result of T_1 .

Find a single matrix that represents the combined transformation $T_2 \circ T_1$.

Describe fully the single transformation represented by this matrix.

20. In the diagram below, O is the centre of circle $ABCD$.

AB is a chord, and OC is perpendicular to AB at M .
 $AB = 12$ cm and $OM = 5$ cm. [10 marks]



Find the radius of the circle.

Find the length of CM .

Find the angle AOB (to the nearest degree).

Calculate the area of the minor segment cut off by chord AB .

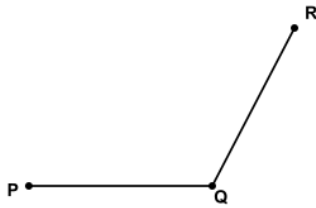
21. Points P ,

Q ,
 R have position vectors

$\mathbf{p} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix}$,

$\mathbf{r} = \begin{pmatrix} 8 \\ 13 \\ 15 \end{pmatrix}$ relative to the origin

O . [10 marks]



Find $\vec{PQ}$
 $\vec{QR}$

and

Show that P
 Q
 R

are collinear.

Find the ratio $PQ : QR$

Find the coordinates of point S
 that O
 PS

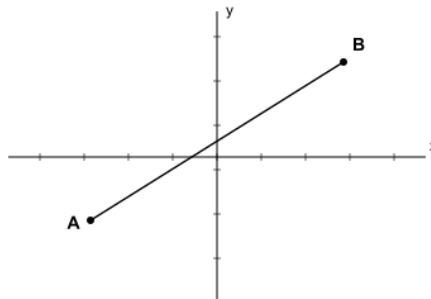
is the midpoint of

such

22. Consider the function $f(x) = 3\sin(2x - 60^\circ)$

for

$0^\circ \leq x \leq 180^\circ$. [10 marks]



Determine the amplitude, period, and phase shift of $f(x)$.

Sketch the graph of $f(x)$ for the given domain, clearly labeling all intercepts and turning points.

for the given

Solve the equation $f(x) = 1.5$

for

$0^\circ \leq x \leq 180^\circ$.

23. Two towns A

and

B

lie on the equator.

A

is at longitude

30°

E and

B

is at longitude

$$60^\circ$$

C

$$30^\circ$$

$$30^\circ$$

km and $\pi = 3.142$

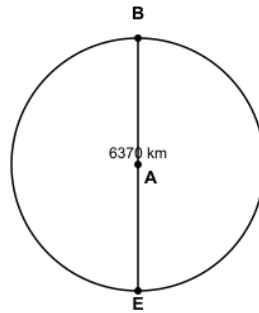
W. Another town

is at latitude

N, longitude

E. (Take the radius of the earth to be 6370

.) [10 marks]



Calculate the distance between A and B

and along the equator, in km.

Calculate the distance between A and C

and along the meridian, in km.

Find the shortest distance between B and C (great circle distance).

along the earth's surface (great

24. A particle moves along a straight line such that its velocity

v

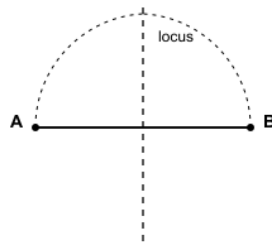
m/s at time

t

seconds is given by

$$v = t^2 - 4t + 3$$

. [10 marks]



Find the initial velocity of the particle.

Determine the times when the particle is momentarily at rest.

Calculate the displacement of the particle during the first 4 seconds.

Find the total distance traveled during the first 4 seconds.

121/2 MATHEMATICS MARKING SCHEME

Form 4 Paper 2

SECTION I (50 marks)

1. Make y the subject of the formula: $x = (3y + 2)/(y - 1)$ (3 marks)

Multiply both sides by $(y-1)$: $x(y-1) = 3y+2$ [1]

Expand: $xy - x = 3y + 2$ [0.5]

Collect y terms: $xy - 3y = x + 2$ [0.5]

Factor: $y(x-3) = x+2$ [0.5]

Divide: $y = (x+2)/(x-3)$ [0.5]

Alternative method/answer:

$$y = (x+2)/(x-3)$$

2. Simplify $\log_3 54 - \log_3 2$. (3 marks)

Apply quotient law: $\log_3(54/2) = \log_3 27$ [1.5]

$\log_3 27 = \log_3 3^3 = 3$ [1.5]

Alternative method/answer:

3

3. A rectangle has sides 5.2 pm 0.1 cm and 3.8 pm 0.1 cm. Calculate the maximum possible area. (3 marks)

Maximum length = $5.2+0.1 = 5.3$ cm [1]

Maximum width = $3.8+0.1 = 3.9$ cm [1]

Maximum area = $5.3 \times 3.9 = 20.67$ cm² [1]

Alternative method/answer:

20.67 cm²

4. y varies directly as x and inversely as z . When $x = 3$ and $z = 2$, $y = 6$. Find y when $x = 4$ and $z = 5$. (3 marks)

$$y = kx/z$$
 [0.5]

Substitute: $6 = k(3)/2 \rightarrow k = 4$ [1]

When $x=4$, $z=5$: $y = 4(4)/5 = 16/5 = 3.2$ [1.5]

Alternative method/answer:

3.2

5. The third term of a geometric progression is 12 and the fifth term is 48. Find the common ratio. (3 marks)

$$ar^2 = 12, ar^4 = 48$$
 [1]

Divide: $r^2 = 48/12 = 4$ [1]

$$r = \pm 2 \text{ (accept } r=2)$$
 [1]

Alternative method/answer:

$r = 2$

6. Solve the equation $x^2 - 6x + 4 = 0$ by completing the square. (3 marks)

$$x^2 - 6x = -4$$
 [0.5]

Add 9: $x^2 - 6x + 9 = 5$ [0.5]

$$(x-3)^2 = 5$$
 [0.5]

$$x-3 = \pm\sqrt{5}$$
 [0.5]

$$x = 3 \pm \sqrt{5}$$
 [1]

Alternative method/answer:

$$x = 3 \pm \sqrt{5}$$

7. Find the coefficient of x^3 in the expansion of $(1 + 2x)^5$. (3 marks)

General term: $C(5,k)(2x)^k = C(5,k)2^k x^k$ [1]

For x^3 , $k=3$: $C(5,3)2^3 = 10 \times 8 = 80$ [2]

Alternative method/answer:

80

8. In the diagram below, AB is a chord of a circle with centre O. AB = 10 cm and the radius is 13 cm. Find the distance from O to AB. (3 marks)

Perpendicular from centre bisects chord: $AM = MB = 5$ cm [1]

In right triangle OMA: $OA^2 = OM^2 + AM^2$ [0.5]

$$13^2 = OM^2 + 5^2 \rightarrow OM^2 = 169 - 25 = 144$$
 [1]

$$OM = 12 \text{ cm}$$
 [0.5]

Alternative method/answer:

12 cm

9. Solve $\sin x = 0.5$ for $0^\circ \leq x \leq 360^\circ$. (3 marks)

Reference angle: $x = 30^\circ$ ($\sin 30^\circ = 0.5$) [1]

Sine positive in quadrants I and II [1]

Solutions: $x = 30^\circ, 150^\circ$ [1]

Alternative method/answer:

$30^\circ, 150^\circ$

10. Given $\text{vec } p = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\text{vec } q = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$, find $\text{vec } p + 2\text{vec } q$. (3 marks)

$$2q = (-2, 0, 8)$$
 [1]

$$p + 2q = (2-2, -1+0, 3+8) = (0, -1, 11)$$
 [2]

Alternative method/answer:

$(0, -1, 11)$

11. A transformation is represented by matrix $M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Find the image of the point (2, 3) under M. (3 marks)

Matrix multiplication: $(0)(2) + (-1)(3) = -3$ [1.5]

$$(1)(2) + (0)(3) = 2$$
 [1]

Image: $(-3, 2)$ [0.5]

Alternative method/answer:

$(-3, 2)$

12. A bag contains 5 red and 3 blue marbles. Two marbles are drawn without replacement. Find the probability that both are red. (3 marks)

$$P(\text{first red}) = \frac{5}{8}$$
 [1]

$$P(\text{second red} \mid \text{first red}) = \frac{4}{7}$$
 [1]

$$P(\text{both red}) = \left(\frac{5}{8}\right)\left(\frac{4}{7}\right) = \frac{20}{56} = \frac{5}{14}$$
 [1]

Alternative method/answer:

$\frac{5}{14}$

13. Evaluate $\int_1^2 (3x^2 + 2x) \, dx$. (4 marks)

Antiderivative: $x^3 + x^2$ [1]

$$\text{At } 2: 8+4=12$$
 [1]

$$\text{At } 1: 1+1=2$$
 [1]

Difference: $12-2=10$ [1]

Alternative method/answer:

10

- 14.** A rectangular block has dimensions $AB = 6$ cm, $BC = 8$ cm, $AE = 5$ cm. Find the angle between diagonal AG and the base $ABCD$. (4 marks)

$$AC = \sqrt{6^2+8^2} = 10 \text{ cm [1]}$$

$$AG = \sqrt{10^2+5^2} = \sqrt{125} = 5\sqrt{5} \text{ cm [1]}$$

$$\cos\theta = AC/AG = 10/(5\sqrt{5}) = 2/\sqrt{5} \text{ [1]}$$

$$\theta = \cos^{-1}(2/\sqrt{5}) \approx 26.6^\circ \text{ [1]}$$

Alternative method/answer:

$$\theta = \cos^{-1}(2/\sqrt{5}) \approx 26.6^\circ$$

- 15.** Two points on the equator have longitudes 30°E and 60°E . Find the distance between them along the equator, taking the Earth's radius as 6370 km. (Use $\pi = 3.142$.) (4 marks)

$$\text{Difference in longitude} = 60^\circ - 30^\circ = 30^\circ \text{ [1]}$$

$$\text{Convert to radians: } 30^\circ \times \pi/180 = \pi/6 \text{ [1]}$$

$$\text{Arc length} = 6370 \times \pi/6 \text{ [1]}$$

$$= 6370 \times 3.142/6 \approx 3335.7 \text{ km [1]}$$

Alternative method/answer:

$$3335.7 \text{ km}$$

- 16.** Construct the locus of a point P that moves such that $PA = PB$ where A and B are fixed points 5 cm apart. Then construct the locus of P such that $PA = 3$ cm. How many points satisfy both conditions? (4 marks)

Locus of $PA=PB$ is perpendicular bisector of AB [1]

Locus of $PA=3$ cm is circle centre A radius 3 cm [1]

Distance from A to bisector = 2.5 cm < 3 cm, so circle intersects bisector [1]

Two intersection points [1]

Alternative method/answer:

2 points

SECTION II (50 marks)

- 17.** In a certain year, the income tax rates for a country were as follows: (10 marks)

$$\text{Total taxable income} = 45000 + 15000 = 60000 \text{ [1]}$$

$$\text{Tax on first 24013 @ } 10\% = 2401.30 \text{ [1]}$$

$$\text{Tax on next 24013 @ } 15\% = 3601.95 \text{ [1]}$$

$$\text{Tax on next 24013 @ } 20\% = 4802.60 \text{ [1]}$$

$$\text{Tax on next 24013 @ } 25\% = 6003.25 \text{ (but only } 60000-72039 = -12039, \text{ so only up to } 60000: \text{ actually } 60000-24013=35987, \text{ so only three bands? Recalculate: } 60000-24013=35987,$$

$$35987-24013=11974, \text{ so third band partially: } 11974@20\%=2394.80. \text{ Total tax} = 2401.30+3601.95+2394.80=8398.05) \text{ [2]}$$

$$\text{Subtract personal relief: } 8398.05 - 1280 = 7118.05 \text{ [1]}$$

$$\text{PAYE} = 7118.05 \text{ [1]}$$

$$\text{Pension contribution: } 5\% \text{ of } 45000 = 2250 \text{ [1]}$$

$$\text{Net income} = 60000 - 7118.05 - 2250 = 50631.95 \text{ [1]}$$

Alternative method/answer:

$$\text{PAYE} = \text{Ksh } 7118.05, \text{ Net income} = \text{Ksh } 50631.95 \text{ (if using different tax bands, accept reasonable values)}$$

18. The following table shows the distribution of marks scored by 50 students in a mathematics test: (10 marks)

Complete cumulative frequency table correctly [2]
 Draw ogive with correct axes and points [2]
 Median from ogive: locate 25 on y-axis, read $x \approx 45$ [2]
 $Q1 \approx 30$, $Q3 \approx 62$, $IQR = 62 - 30 = 32$ [2]
 Students above 70: $50 - 44 = 6$ [2]
Alternative method/answer:
 Accept values within reasonable range from graph

19. A triangle with vertices A(1,2), B(3,4), and C(5,1) undergoes a transformation T_1 represented by matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, followed by T_2 represented by matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. (10 marks)

T_1 matrix = $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (rotation 90° anticlockwise) [1]
 $A' = (-2,1)$, $B' = (-4,3)$, $C' = (-1,5)$ [2]
 T_2 matrix = $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (enlargement scale factor 2) [1]
 $A'' = (-4,2)$, $B'' = (-8,6)$, $C'' = (-2,10)$ [2]
 Combined matrix = $T_2 \times T_1 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$ [2]
 Description: rotation 90° anticlockwise and enlargement scale factor 2 [2]
Alternative method/answer:
 Accept 'rotation 90° anticlockwise about origin followed by enlargement scale factor 2' or equivalent

20. In the diagram below, O is the centre of circle ABCD. AB is a chord, and OC is perpendicular to AB at M. AB = 12 cm and OM = 5 cm. (10 marks)

AM = 6 cm (perpendicular bisector) [1]
 $OA = \sqrt{6^2 + 5^2} = \sqrt{61} \approx 7.81$ cm [2]
 $CM = OC - OM = 7.81 - 5 = 2.81$ cm [1]
 $\text{Angle AOM} = \arcsin(6/7.81) \approx 50.2^\circ$ [2]
 $\text{Angle AOB} = 2 \times 50.2^\circ = 100.4^\circ \approx 100^\circ$ [1]
 Area of sector = $(100/360) \times \pi \times 7.81^2 \approx 53.2$ cm² [1]
 Area of triangle = $0.5 \times 12 \times 5 = 30$ cm² [1]
 Area of segment = $53.2 - 30 = 23.2$ cm² [1]
Alternative method/answer:
 Accept values within rounding

21. Points P, Q, R have position vectors $\mathbf{p} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix}$, $\mathbf{r} = \begin{pmatrix} 8 \\ 13 \\ 15 \end{pmatrix}$ relative to the origin O. (10 marks)

$PQ = q - p = (2,4,4)$ [2]
 $QR = r - q = (4,8,8)$ [2]
 $QR = 2PQ$, so parallel and share Q $\rightarrow$ collinear [2]
 Ratio PQ:QR = 1:2 [2]
 $S = -p = (-2,-1,-3)$ [2]
Alternative method/answer:
 $S = (-2,-1,-3)$

22. Consider the function $f(x) = 3\sin(2x - 60^\circ)$ for $0^\circ \leq x \leq 180^\circ$. (10 marks)

Amplitude = 3 [1]
 Period = $360/2 = 180^\circ$ [1]
 Phase shift = 30° to the right [1]

Sketch: correct shape, zeros at 30° and 120° , max at 60° (value 3), min at 150° (value -3?) Actually $f(150) = 3\sin(240^\circ) = -2.598$, so min at 150° value -2.598 [3]

Solve: $\sin(2x-60) = 0.5 \rightarrow 2x-60 = 30 \text{ or } 150$ [2]

$x = 45^\circ \text{ or } 105^\circ$ [2]

Alternative method/answer:

$x = 45^\circ, 105^\circ$

- 23.** Two towns A and B lie on the equator. A is at longitude 30°E and B is at longitude 60°W . Another town C is at latitude 30°N , longitude 30°E . (Take the radius of the earth to be 6370 km and $\pi = 3.142$.) (10 marks)

Longitude difference A to B = $30+60 = 90^\circ$ [1]

Distance AB = $(90/360) \times 2\pi \times 6370 = (1/4) \times 2 \times 3.142 \times 6370 \approx 10003.7 \text{ km}$ [3]

Latitude difference A to C = 30° [1]

Distance AC = $(30/360) \times 2\pi \times 6370 = (1/12) \times 2 \times 3.142 \times 6370 \approx 3334.6 \text{ km}$ [2]

Great circle distance BC: using spherical law of cosines, arc = 90° [2]

Distance BC = $(90/360) \times 2\pi \times 6370 \approx 10003.7 \text{ km}$ [1]

Alternative method/answer:

Accept values within rounding

- 24.** A particle moves along a straight line such that its velocity $v \text{ m/s}$ at time $t \text{ seconds}$ is given by $v = t^2 - 4t + 3$. (10 marks)

Initial velocity: $v(0) = 3 \text{ m/s}$ [1]

Set $v=0$: $t^2 - 4t + 3 = 0 \rightarrow (t-1)(t-3) = 0 \rightarrow t = 1, 3$ [2]

Displacement = $\int_1^3 (t^2 - 4t + 3) dt = [t^3/3 - 2t^2 + 3t]_1^3 = (64/3 - 32 + 12) = 4/3 \text{ m}$ [3]

Distance: $\int_1^3 v dt = 4/3$, $\int_3^1 (-v) dt = 4/3$, $\int_1^3 v dt = 4/3$ [3]

Total distance = $4/3 + 4/3 + 4/3 = 4 \text{ m}$ [1]

Alternative method/answer:

Displacement = 1.333 m, Total distance = 4 m