

EDUZEDCO
121/2 MATHEMATICS
Paper 2
Form 1 End Term Examination

Time: 2 ½ Hours

Name: Adm No:
 School: Class:
 Signature: Date:

INSTRUCTIONS TO CANDIDATES

- (a) Write your name and index number in the spaces provided at the top of this page.
- (b) Write your school name, sign and write the date of the examination in the spaces provided above.
- (c) This paper consists of **Two** sections: **Section I** and **Section II**.
- (d) Answer **ALL** questions in Section I and any **five questions** from Section II.
- (e) Marks may be given for correct working, relevant points, and clear presentation.
- (f) Non-programmable silent electronic calculators may be used where necessary.
- (g) KNEC Mathematical Tables may be used.
- (h) This paper consists of **8 printed pages**.
- (i) Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.
- (j) Candidates should answer the questions in English.

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total	Grand Total

SECTION I

1. Make x

$$y = 3x + 5$$

2. Simplify $\sqrt{50} + \sqrt{18}$

3. A length is measured as 12.5
nearest 0.1

4. y

$$x$$

$$z$$

$$y = 8$$

$$x = 4$$

$$z = 2$$

$$y$$

$$x = 6$$

$$z = 3$$

5. The first term of a geometric progression is

$$2$$

$$3$$

$$5$$

6. Solve the equation $x^2 + 6x + 9 = 25$

7. Expand $(1 + 2x)^4$
 x^2

8. In the diagram, O

circle. AB

$$10$$

$$OM$$

$$AB$$

$$OM = 3$$

the subject of the formula

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cm to the

cm. Find the percentage error.

varies directly as

and inversely as the square of

. Given that

when

and

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and the common ratio is

. Find the sum of the first
terms.

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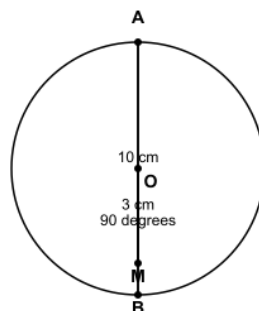
is a chord of length

cm and

is perpendicular to

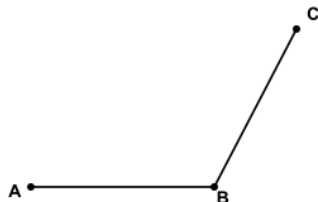
with

cm. Find the radius of the circle.



9. Solve the equation $\sin \theta = 0.5$ for $0^\circ \leq \theta \leq 360^\circ$.

10. Given vectors $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, find $\vec{a} \cdot \vec{b}$.

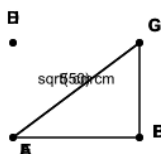


11. Find the inverse of the matrix $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$.

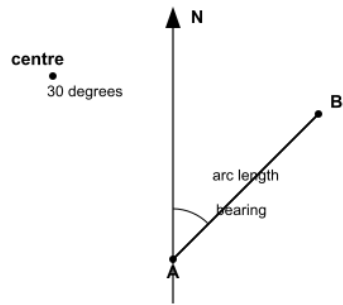
12. A bag contains 3 red balls and 5 blue balls. Two balls are drawn without replacement. Find the probability that both balls are red.

13. Evaluate $\int_1^3 (2x + 1) dx$.

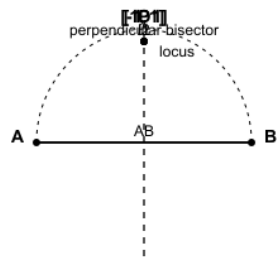
14. A rectangular box has dimensions 4 cm, 3 cm, and 5 cm. Find the angle between the diagonal of the base and the body diagonal.



15. Two points on the equator have longitudes 30° E and 60° E. Find the distance between them in kilometres. (Take radius of Earth $= 6370$ km and $\pi = 22/7$).



16. Construct the locus of a point P that is equidistant from points A and B and also equidistant from lines l_1 and l_2 that intersect at O . Describe the intersection of these two loci.



SECTION II

Answer any five questions from this section.

17. In a certain country, income tax is charged on annual taxable income as follows:

First Ksh 120,000:	10%
Next Ksh 100,000:	15%
Next Ksh 150,000:	20%
Next Ksh 200,000:	25%
Amount above Ksh 570,000:	30%

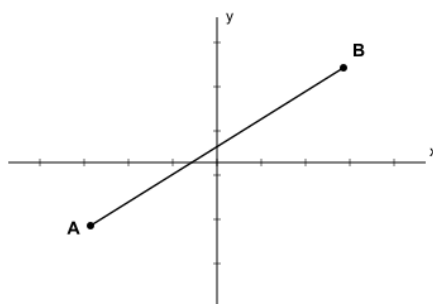
 A person earns a monthly salary of Ksh 60,000. He is entitled to a monthly personal relief of Ksh 1,200. [10 marks]

- (a) Calculate his annual taxable income.
- (b) Calculate the total income tax he pays per year.
- (c) Determine his net monthly income after tax.

18. The following data shows the marks of 40 students in a mathematics test.

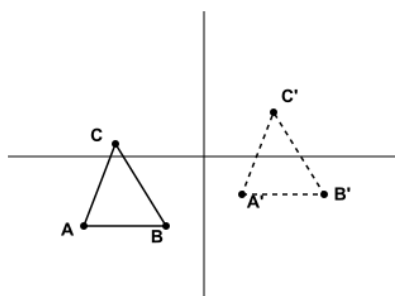
Marks	10-19	20-29	30-39	40-49	50-59	60-69	70-79
Frequency	2	6	10	12	6	3	1

 [10 marks]



- (a) Construct a cumulative frequency table.
- (b) Draw a cumulative frequency curve (ogive).
- (c) Using your ogive, estimate the median mark.
- (d) Determine the interquartile range.

19. A transformation T is represented by the matrix $\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$. [10 marks]



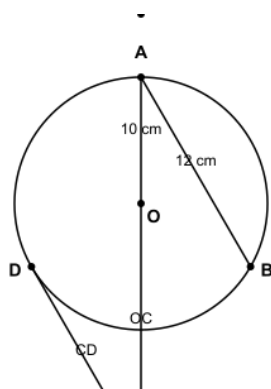
(a) Find the image of the point $P(3, -2)$ under T .

(b) The image of a point Q under T is

$Q'(5, 4)$. Find the coordinates of Q .

(c) A triangle with vertices $A(1, 0)$, $B(2, 3)$, $C(4, 1)$ is transformed by T . Find the area of the image triangle.

20. In the diagram below, O is the center of the circle. AB is a chord, OC is perpendicular to AB at C , and $AB = 12$ cm. The radius of the circle is 10 cm. [10 marks]

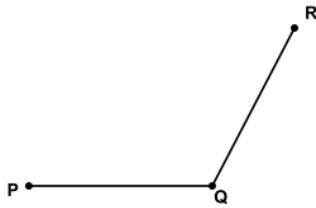


(a) Find the length of OC .

(b) Find the length of the chord CD .

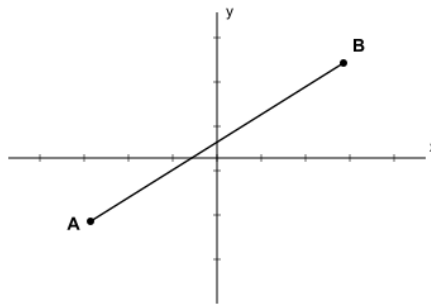
AB is parallel to CD and at a distance of 6 cm from O .

21. Given points $P(1, 2, 3)$, $Q(4, 5, 6)$, $R(7, 8, 9)$, $S(2, 4, 6)$.



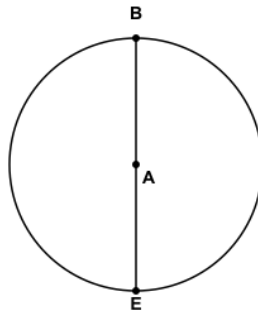
- (a) Find the vectors $\vec{PQ}$ and $\vec{PR}$.
- (b) Show that points P , Q , and R are collinear.
- (c) Find the ratio in which S divides PQ .

22. [10 marks]



- (a) On the same axes, sketch the graphs of $y = \sin x$ and $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$.
- (b) State the amplitude and period of each function.
- (c) Determine the range of values of x for which $\sin x \geq \cos x$ in the interval $0^\circ \leq x \leq 360^\circ$.

23. Two points A and B on the earth's surface have coordinates $A(30^\circ \text{ N}, 45^\circ \text{ E})$ and $B(30^\circ \text{ N}, 135^\circ \text{ W})$. Take the earth's radius as 6370 km. [10 marks]



(a) Find the difference in longitude between

A

and

B

.

(b) Calculate the distance between A

and B

along the parallel of latitude.

(c) Find the shortest distance between A

and B

along the great circle.

24. A particle moves along a straight line such that its displacement

s

meters from a fixed point

O

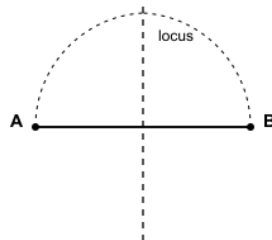
after

t

seconds is given by

$$s = t^3 - 6t^2 + 9t$$

. [10 marks]



(a) Find the initial velocity of the particle.

(b) Determine the time when the particle is momentarily at rest.

(c) Calculate the total distance traveled in the first 4 seconds.

121/2 MATHEMATICS MARKING SCHEME

Form 1 Paper 2

SECTION I (50 marks)

1. Make x the subject of the formula $y = 3x + 5$. (2 marks)

Subtract 5 from both sides: $y - 5 = 3x$ [1]

Divide both sides by 3: $x = (y - 5)/3$ [1]

Alternative method/answer:

$$x = (y-5)/(3)$$

2. Simplify $\sqrt{50} + \sqrt{18}$. (3 marks)

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2} \text{ [1]}$$

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2} \text{ [1]}$$

$$5\sqrt{2} + 3\sqrt{2} = 8\sqrt{2} \text{ [1]}$$

Alternative method/answer:

$$8\sqrt{2}$$

3. A length is measured as 12.5 cm to the nearest 0.1 cm. Find the percentage error. (2 marks)

$$\text{Absolute error} = 0.05 \text{ cm [1]}$$

$$\text{Percentage error} = (0.05)/(12.5) \times 100\% = 0.4\% \text{ [1]}$$

Alternative method/answer:

$$0.4\%$$

4. y varies directly as x and inversely as the square of z . Given that $y = 8$ when $x = 4$ and $z = 2$, find y when $x = 6$ and $z = 3$. (4 marks)

$$\text{Write variation equation: } y = k(x)/(z^2) \text{ [1]}$$

$$\text{Substitute given: } 8 = k(4)/(2^2) \text{ Rightarrow } 8 = k(4)/(4) \text{ Rightarrow } k = 8 \text{ [1]}$$

$$\text{New equation: } y = 8(6)/(3^2) \text{ [1]}$$

$$y = 8 \times (6)/(9) = (48)/(9) = (16)/(3) \text{ [1]}$$

Alternative method/answer:

$$(16)/(3) \text{ or } 5(1)/(3)$$

5. The first term of a geometric progression is 2 and the common ratio is 3. Find the sum of the first 5 terms. (3 marks)

$$\text{Sum formula: } S_n = a(r^n - 1)/(r - 1) \text{ [1]}$$

$$S_5 = 2(3^5 - 1)/(3 - 1) \text{ [1]}$$

$$= 2(243 - 1)/(2) = 2 \times (242)/(2) = 242 \text{ [1]}$$

Alternative method/answer:

$$242$$

6. Solve the equation $x^2 + 6x + 9 = 25$. (3 marks)

$$\text{Recognize perfect square: } x^2 + 6x + 9 = (x+3)^2 \text{ [1]}$$

$$\text{Set } (x+3)^2 = 25 \text{ [1]}$$

$$x+3 = \pm 5 \text{ Rightarrow } x = 2 \text{ or } x = -8 \text{ [1]}$$

Alternative method/answer:

$$x = 2 \text{ or } x = -8$$

7. Expand $(1 + 2x)^4$ up to the term in x^2 . (3 marks)

$$\text{Binomial expansion: } (1+2x)^4 = \text{binom}{4}{0}(2x)^0 + \text{binom}{4}{1}3(2x)^1 + \text{binom}{4}{2}(2x)^2 + \text{ldots} \text{ [1]}$$

$$= 1 + 4(2x) + 6(4x^2) + \text{ldots} \text{ [1]}$$

$$= 1 + 8x + 24x^2 \text{ [1]}$$

Alternative method/answer:

$$1 + 8x + 24x^2$$

- 8.** In the diagram, O is the centre of the circle. AB is a chord of length 10 cm and OM is perpendicular to AB with OM = 3 cm. Find the radius of the circle. (4 marks)

Since OM perp AB, M is midpoint of AB, so AM = 5 cm [1]

In right triangle OMA, $OA^2 = OM^2 + AM^2$ [1]

$$OA^2 = 3^2 + 5^2 = 9 + 25 = 34 \text{ [1]}$$

Radius OA = $\sqrt{34}$ cm [1]

Alternative method/answer:

$$\sqrt{34} \text{ cm}$$

- 9.** Solve the equation $\sin \theta = 0.5$ for $0^\circ \leq \theta \leq 360^\circ$. (4 marks)

Reference angle: $\sin^{-1}(0.5) = 30^\circ$ [1]

Sine positive in first and second quadrants [1]

$$\theta = 30^\circ \text{ [1]}$$

$$\theta = 180^\circ - 30^\circ = 150^\circ \text{ [1]}$$

Alternative method/answer:

$$\theta = 30^\circ, 150^\circ$$

- 10.** Given vectors $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, find $\vec{a} \cdot \vec{b}$. (4 marks)

Dot product formula: $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$ [1]

$$= (1)(4) + (2)(5) + (3)(6) \text{ [1]}$$

$$= 4 + 10 + 18 \text{ [1]}$$

$$= 32 \text{ [1]}$$

Alternative method/answer:

$$32$$

- 11.** Find the inverse of the matrix $\begin{pmatrix} 2 & 4 \\ -3 & 1 \end{pmatrix}$. (4 marks)

Determinant: $2 \times 1 - 3 \times 4 = 8 - 12 = -4$ [1]

Inverse formula: $\frac{1}{(\det)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ [1]

$$= \frac{1}{(-4)} \begin{pmatrix} 1 & -4 \\ 3 & 2 \end{pmatrix} \text{ [1]}$$

$$= \begin{pmatrix} -1/4 & 1 \\ -3/4 & -1/2 \end{pmatrix} \text{ [1]}$$

Alternative method/answer:

$$\begin{pmatrix} -0.25 & 1 \\ -0.75 & -0.5 \end{pmatrix}$$

- 12.** A bag contains 3 red balls and 5 blue balls. Two balls are drawn without replacement. Find the probability that both balls are red. (4 marks)

$$P(\text{first red}) = \frac{3}{8} \text{ [1]}$$

$$P(\text{second red} \mid \text{first red}) = \frac{2}{7} \text{ [1]}$$

$$P(\text{both red}) = \frac{3}{8} \times \frac{2}{7} \text{ [1]}$$

$$= \frac{6}{56} = \frac{3}{28} \text{ [1]}$$

Alternative method/answer:

$$\frac{3}{28}$$

- 13.** Evaluate $\int_1^3 (2x + 1) \, dx$. (3 marks)

Integral: $\int (2x+1) \, dx = x^2 + x$ [1]

Evaluate at limits: $(3^2 + 3) - (1^2 + 1)$ [1]

$$= (9+3) - (1+1) = 12 - 2 = 10 \text{ [1]}$$

Alternative method/answer:

- 14.** A rectangular box has dimensions 4 cm, 3 cm, and 5 cm. Find the angle between the diagonal of the base and the body diagonal. (4 marks)

Base diagonal $AC = \sqrt{4^2 + 3^2} = 5 \text{ cm}$ [1]

Body diagonal $AG = \sqrt{4^2 + 3^2 + 5^2} = \sqrt{50} = 5\sqrt{2} \text{ cm}$ [1]

Angle: $\cos \theta = (AC)/(AG) = (5)/(5\sqrt{2}) = (1)/(\sqrt{2})$ [1]

$\theta = \cos^{-1}(1/\sqrt{2}) = 45^\circ$ [1]

Alternative method/answer:

45°

- 15.** Two points on the equator have longitudes 30°E and 60°E . Find the distance between them in kilometres. (Take radius of Earth = 6370 km and $\pi = 22/7$). (3 marks)

Longitude difference = $60^\circ - 30^\circ = 30^\circ$ [1]

Distance = $(30)/(360) \times 2\pi \times 6370$ [1]

$= (1)/(12) \times 2 \times (22)/(7) \times 6370 = 3336(2)/(3) \text{ km}$ [1]

Alternative method/answer:

$3336(2)/(3) \text{ km}$ or 3336.67 km

- 16.** Construct the locus of a point P that is equidistant from points A and B and also equidistant from lines l_1 and l_2 that intersect at O. Describe the intersection of these two loci. (4 marks)

Locus equidistant from A and B is the perpendicular bisector of AB [1]

Locus equidistant from lines l_1 and l_2 is the angle bisector of the angle between them [1]

The intersection is the point(s) where the perpendicular bisector meets the angle bisector [2]

Alternative method/answer:

The intersection is the point(s) where the perpendicular bisector of AB meets the angle bisector of the lines.

SECTION II (50 marks)

- 17.** In a certain country, income tax is charged on annual taxable income as follows: (10 marks)
- beginitemize \item First Ksh 120,000: 10% \item Next Ksh 100,000: 15% \item Next Ksh 150,000: 20% \item Next Ksh 200,000: 25% \item Amount above Ksh 570,000: 30% enditemize
- A person earns a monthly salary of Ksh 60,000. He is entitled to a monthly personal relief of Ksh 1,200.

Annual salary = $60,000 \times 12 = 720,000$ [1]

Annual personal relief = $1,200 \times 12 = 14,400$ [1]

Annual taxable income = $720,000 - 14,400 = 705,600$ [1]

First 120,000 $\times 10\% = 12,000$ [1]

Next 100,000 $\times 15\% = 15,000$ [1]

Next 150,000 $\times 20\% = 30,000$ [1]

Next 200,000 $\times 25\% = 50,000$ [1]

Remaining 135,600 $\times 30\% = 40,680$ [1]

Total tax = $12,000 + 15,000 + 30,000 + 50,000 + 40,680 = 147,680$ [1]

Net monthly income = $(720,000 - 147,680)/12 = 47,693.33$ [1]

Alternative method/answer:

Annual taxable income = Ksh 705,600; Total tax = Ksh 147,680; Net monthly income = Ksh 47,693.33

18. The following data shows the marks of 40 students in a mathematics test. (10 marks)

Marks	10-19	20-29	30-39	40-49	50-59	60-69	70-79
Frequency	2	6	10	12	6	3	1

Cumulative frequencies: 2, 8, 18, 30, 36, 39, 40 [2]
 Upper class boundaries: 19.5, 29.5, 39.5, 49.5, 59.5, 69.5, 79.5 [1]
 Plot points and draw smooth ogive [2]
 Median (20th value) read from ogive ≈ 42 [2]
 Lower quartile (10th value) ≈ 32 , Upper quartile (30th value) ≈ 52 [2]
 Interquartile range = $52 - 32 = 20$ [1]
Alternative method/answer:
 Median ≈ 42 ; Interquartile range = 20

19. A transformation T is represented by the matrix $\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$ (10 marks)

(a) $T(P) = (2 \times 3 + (-1) \times (-2), 1 \times 3 + 0 \times (-2)) = (6+2, 3+0) = (8,3)$ [3]
 (b) Let $Q=(x,y)$. Then $2x - y = 5$ and $x = 4$ [2]
 $x=4$, then $2(4)-y=5 \Rightarrow 8-y=5 \Rightarrow y=3$, so $Q=(4,3)$ [2]
 (c) Area of original triangle = 4 (using shoelace or formula) [1]
 Determinant of T = $2 \times 0 - (-1) \times 1 = 1$ [1]
 Area of image = $4 \times |1| = 4$ square units [1]
Alternative method/answer:
 (a) (8,3); (b) (4,3); (c) 4 square units

20. In the diagram below, O is the center of the circle. AB is a chord, OC is perpendicular to AB at C, and AB = 12 cm. The radius of the circle is 10 cm. (10 marks)

(a) Since OC $\perp$ AB, C is midpoint, so AC = 6 cm [1]
 In right triangle OAC, OA=10, AC=6 [1]
 $OC = \sqrt{(10^2 - 6^2)} = \sqrt{(100-36)} = \sqrt{64} = 8$ cm [2]
 (b) Distance from O to CD is 6 cm, let E be midpoint of CD [1]
 In right triangle OED, OD=10, OE=6 [1]
 $ED = \sqrt{(10^2 - 6^2)} = 8$ cm [2]
 $CD = 2 \times ED = 16$ cm [2]
Alternative method/answer:
 (a) OC = 8 cm; (b) CD = 16 cm

21. Given points P(1,2,3), Q(4,5,6), R(7,8,9), and S(2,4,6). (10 marks)

(a) $PQ = (4-1, 5-2, 6-3) = (3,3,3)$ [2]
 $PR = (7-1, 8-2, 9-3) = (6,6,6)$ [2]
 (b) $PR = 2 \times PQ$, so vectors are parallel and share point P, hence collinear [2]
 (c) Let S divide PQ in ratio k:1. Then $(4k+1)/(k+1)=2 \Rightarrow 4k+1=2k+2 \Rightarrow 2k=1 \Rightarrow k=0.5$ [2]
 Ratio = 0.5:1 = 1:2 (from P to Q) [2]
Alternative method/answer:
 (a) $PQ=(3,3,3)$, $PR=(6,6,6)$; (b) collinear; (c) 1:2

22. (10 marks)

(a) Sketch $\sin x$ and $\cos x$ for 0° to 360° with correct shapes and intersections [4]
 (b) Amplitude of $\sin x = 1$, period = 360° [1]
 Amplitude of $\cos x = 1$, period = 360° [1]
 (c) Solve $\sin x = \cos x \Rightarrow \tan x = 1 \Rightarrow x = 45^\circ, 225^\circ$ [2]
 $\sin x \geq \cos x$ for $45^\circ \leq x \leq 225^\circ$ [2]

Alternative method/answer:

(a) Sketches; (b) Amplitude 1, period 360° ; (c) $45^\circ \leq x \leq 225^\circ$

23. Two points A and B on the earth's surface have coordinates A(30° N, 45° E) and B(30° N, 135° W). Take the earth's radius as 6370 km. (10 marks)

(a) Longitude difference = $45^\circ + 135^\circ = 180^\circ$ [2]

(b) Radius at latitude 30° N: $r = R \cos 30^\circ = 6370 \times \sqrt{3}/2 \approx 5517.6$ km [2]

Distance along parallel = $(180/360) \times 2\pi r = \pi r \approx 17330$ km [2]

(c) Great circle distance: angular distance = $180^\circ - 30^\circ - 30^\circ = 120^\circ$ [2]

Distance = $(120/360) \times 2\pi R = (1/3) \times 2\pi \times 6370 \approx 13330$ km [2]

Alternative method/answer:

(a) 180° ; (b) ≈ 17330 km; (c) ≈ 13330 km

24. A particle moves along a straight line such that its displacement s meters from a fixed point O after t seconds is given by $s = t^3 - 6t^2 + 9t$. (10 marks)

(a) $v = ds/dt = 3t^2 - 12t + 9$ [2]

At $t=0$, $v = 9$ m/s [1]

(b) Set $v=0$: $3t^2 - 12t + 9 = 0 \Rightarrow t^2 - 4t + 3 = 0 \Rightarrow (t-1)(t-3) = 0$ [2]

$t=1$ s and $t=3$ s [1]

(c) $s(0)=0$, $s(1)=4$, $s(3)=0$, $s(4)=4$ [2]

Total distance = $|s(1)-s(0)| + |s(3)-s(1)| + |s(4)-s(3)| = 4+4+4 = 12$ m [2]

Alternative method/answer:

(a) 9 m/s; (b) $t=1$ s and $t=3$ s; (c) 12 m