

EDUZEDCO
121/2 MATHEMATICS
Paper 2
Form 3 End Term Examination

Time: 2 ½ Hours

Name: Adm No:
 School: Class:
 Signature: Date:

INSTRUCTIONS TO CANDIDATES

- (a) Write your name and index number in the spaces provided at the top of this page.
- (b) Write your school name, sign and write the date of the examination in the spaces provided above.
- (c) This paper consists of **Two** sections: **Section I** and **Section II**.
- (d) Answer **ALL** questions in Section I and any **five questions** from Section II.
- (e) Marks may be given for correct working, relevant points, and clear presentation.
- (f) Non-programmable silent electronic calculators may be used where necessary.
- (g) KNEC Mathematical Tables may be used.
- (h) This paper consists of **8 printed pages**.
- (i) Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.
- (j) Candidates should answer the questions in English.

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total	Grand Total

SECTION I

1. Make r

$$V = \frac{1}{3}\pi r^2 h$$

2. Simplify $\frac{3}{\sqrt{5} - \sqrt{2}}$
denominator.

3. A rectangle measures 8.5 cm
5.2 cm
0.1 cm
area.

4. y

$$x$$

$$x$$

$$x = 2$$

$$y = 14$$

$$x = 3$$

$$y = 30$$

$$y$$

$$x = 4$$

5. The third term of a GP is 12
term is 96
common ratio.

6. Solve the equation $x^2 + 6x + 2 = 0$
square, leaving your answer in surd form.

7. Expand $(2 - x)^5$

$$x$$

$$x^3$$

8. In the diagram below, O
the circle. AB
12 cm
 OX
 X
 $OX = 8$ cm

the subject of the formula

.

by rationalizing the

by

, each measured to the nearest

. Calculate the percentage error in the

varies partly as

and partly as the square of

. When

,

; when

,

. Find

when

.

and the sixth

. Find the first term and the

by completing the

in ascending powers of

up to the term in

.

is the centre of

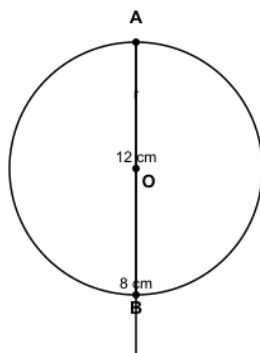
is a chord of length

perpendicular to

at

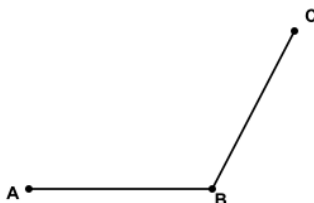
. If

, find the radius of the circle.

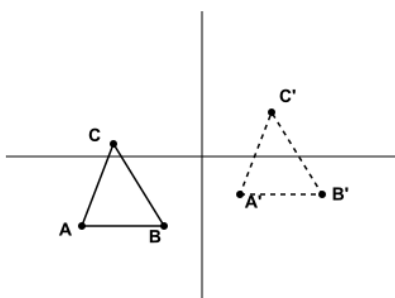


9. Solve the equation $\sin 2\theta = \cos \theta$ for $0^\circ \leq \theta < 360^\circ$.

10. Given vectors $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$, find the magnitude of $2\mathbf{a} - \mathbf{b}$.



11. A transformation is represented by matrix $M = \begin{pmatrix} 2 & 8 \\ -1 & 3 \\ 0 & 0 \end{pmatrix}$. Find the image of the point $P(2, -3)$ under M .



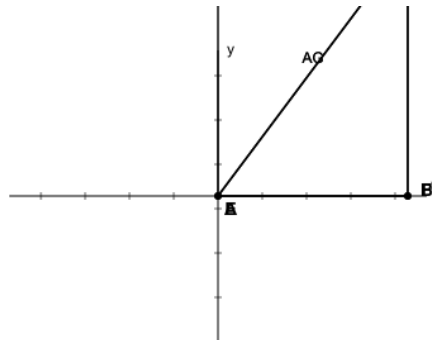
12. A bag contains 4 red and 6 blue marbles. Two marbles are drawn without replacement. Find the probability that both are red.

13. Evaluate $\int_1^2 (3x^2 - 2x + 1) dx$.

14. A rectangular block has dimensions $AB = 6$ cm, $BC = 8$ cm, and $AE = 10$ cm. Find the angle between the diagonal

AG
 $ABCD$

and the base



15. Two points A and B on the Equator have longitudes

30°

E and

45°

W respectively. Calculate the distance

between them along the Equator, taking the Earth's radius as

6370 km

. (Use

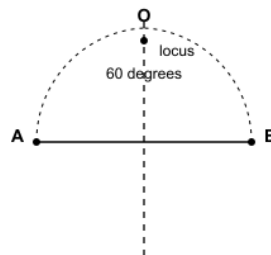
$\pi = 3.142$

)

16. Construct the locus of points equidistant from two intersecting lines forming an angle of

60°

. Describe the locus.



SECTION II

Answer any five questions from this section.

17. In a certain country, income tax is calculated as follows:

Sh. 10, 000	per month: tax rate
10%	
Sh. 20, 000	per month: tax rate
15%	
Sh. 30, 000	per month: tax rate
20%	
Sh. 60, 000	per month: tax rate
25%	

A person earns a monthly salary of Sh. 85, 000. They are entitled to a personal relief of Sh. 1, 200 per month. (a) (4 marks) (b) (2

marks) (c) If the person also contributes 5%

of their salary to a pension scheme which is tax exempt, find the new net tax payable. (4 marks) [10 marks]

Calculate the gross tax payable before relief.

Determine the net tax payable after relief.

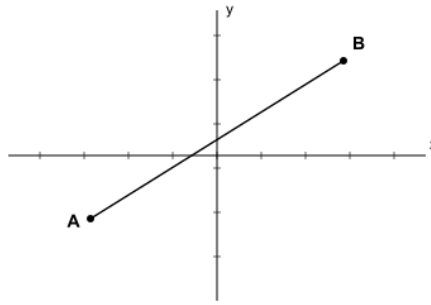
If the person also contributes 5% of their salary to a pension scheme which is tax exempt, find the new net tax payable.

18. The table below shows the distribution of marks scored by 100 students in a Mathematics test.

Marks & Frequency

10 – 19	4
20 – 29	12
30 – 39	20
40 – 49	30
50 – 59	18
60 – 69	10
70 – 79	6

(a) (4 marks) (b) Using your ogive, estimate: (i) The median mark. (2 marks) (ii) The interquartile range. (2 marks) (c) (2 marks) [10 marks]



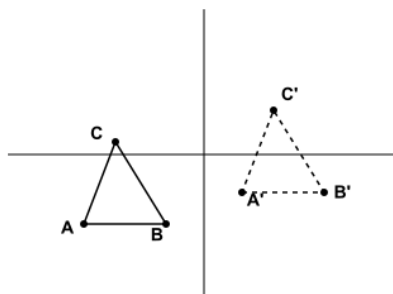
Construct a cumulative frequency table and draw an ogive.

Using your ogive, estimate the median mark.

Using your ogive, estimate the interquartile range.

Determine the variance of the marks.

19. A triangle with vertices $A(1, 2)$, $B(3, 4)$ and $C(5, 1)$ undergoes two successive transformations. First transformation: Rotation through 90° anticlockwise about the origin, represented by matrix R . Second transformation: Reflection in the line $y = x$, represented by matrix M . (a) Write down the matrices R and M . (2 marks) (b) Find the image of point A after the first transformation. (2 marks) (c) Determine the single transformation matrix equivalent to the combined transformation $M \circ R$. (i.e., first R , then M). (3 marks) (d) (3 marks) [10 marks]



Write down the matrices R and M .

Find the image of point A after the first transformation.

Determine the single transformation matrix equivalent to the combined transformation $M \rightarrow R$.

Apply the combined transformation to the triangle and find the coordinates of its image.

20. In the diagram below, O

is the centre of

the circle. AB

is a chord, and

C

is a point on the circle such that

OC

is perpendicular to

AB

. Given that

$AB = 24$

cm and

$OC = 5$

cm. $\begin{array}{l} \text{begincenter} \\ \text{begin tikzpicture} \end{array}$

$\begin{array}{l} [\text{scale}=0.8] \text{draw} (0,0) \text{circle} (13); \text{draw} (-5,-12) -- (5,-12); \text{draw} (0,0) -- (0,-12); \text{draw} (0,0) -- (-5,-12); \text{draw} \\ (0,0) -- (5,-12); \text{node[above]} \text{at} (0,0) \{ O \end{array}$

$\};$

$\text{node[below]} \text{at} (-5,-12) \{ A$

$\}; \text{node[below]}$

$\text{at} (5,-12) \{ B$

$\}; \text{node[right]} \text{at} (0,-6) \{$

C

$\}; \text{end tikzpicture} \text{endcenter} \text{(a) (3 marks)}$

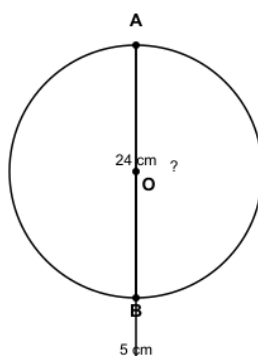
(b) Calculate the angle $\angle AOB$

correct to 1

decimal place. (3 marks) (c) Determine the area of the minor segment

ACB

. (4 marks) [10 marks]



Find the radius of the circle.

Calculate the angle $\angle AOB$ correct to 1 decimal place.

Determine the area of the minor segment ACB .

21. Points P

Q

R

have position vectors

$\vec{p} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{q} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, $\vec{r} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$. (a)

Find the vector $\vec{PQ}$

and

$\vec{QR}$

. (2 marks) (b) Show that

P

,

Q

,

R

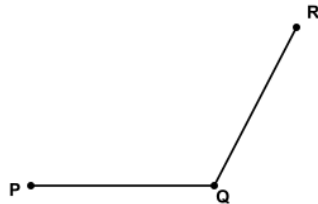
are collinear. (3 marks) (c) Find the ratio

$PQ : QR$

. (2 marks) (d) Determine the coordinates

of point S
 Q
 PS

such that
 is the midpoint of
 . (3 marks) [10 marks]



Find the vector PQ and QR .

Show that P , Q , R are collinear.

Find the ratio $PQ:QR$.

Determine the coordinates of point S such that Q is the midpoint of PS .

22. Consider the function $f(x) = 3\sin(2x) + 1$

for

$0^\circ \leq x \leq 360^\circ$. (a) Determine the amplitude, period, and phase shift of

f

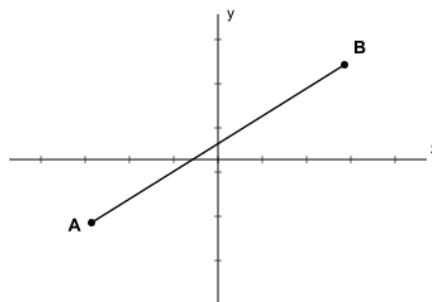
. (3 marks) (b) Sketch the graph of

f

for the given interval, clearly labeling key

points. (4 marks) (c) Solve the equation $f(x) = 2$

for $0^\circ \leq x \leq 360^\circ$. (3 marks) [10 marks]



Determine the amplitude, period, and phase shift of f .

Sketch the graph of f for the given interval, clearly labeling key points.

Solve the equation $f(x) = 2$ for $0^\circ \leq x \leq 360^\circ$.

23. Two towns A

and

B

lie on the equator.

A

is at longitude

30°

E, and

B

is at longitude

60°

6370

A

B

(b) A plane flies from A

C

45°

D

60°

B

marks) [10 marks]

E. Take the radius of the earth as

km. (a) Calculate the distance between

and

along the equator in kilometres. (3 marks)

due north to

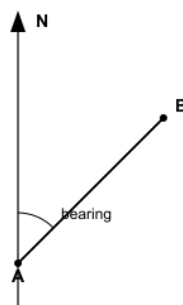
at latitude

N, then due east to

at longitude

E, then due south back to

. Determine the total distance travelled. (7



Calculate the distance between A and B along the equator in kilometres.

Determine the total distance travelled by the plane from A to C to D to B.

24. A particle moves along a straight line with velocity

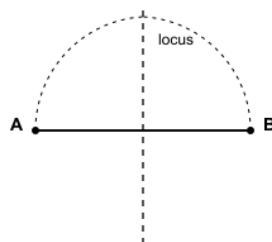
$$v(t) = 3t^2 - 12t + 9$$

t

m/s, where

is time in seconds, $t \geq 0$. (a) (1 mark)

(b) (2 marks) (c) (3 marks) (d) (4 marks) [10 marks]



Find the initial velocity of the particle.

Determine the times when the particle is momentarily at rest.

Calculate the displacement of the particle during the first 4 seconds.

Find the total distance traveled during the first 4 seconds.

121/2 MATHEMATICS MARKING SCHEME

Form 3 Paper 2

SECTION I (50 marks)

1. Make r the subject of the formula $V = \frac{1}{3}\pi r^2 h$. (2 marks)

Multiply both sides by 3: $3V = \pi r^2 h$ [1]

Divide by πh and take square root: $r = \sqrt{3V/(\pi h)}$ [1]

Alternative method/answer:

Accept $r = \pm \sqrt{3V/(\pi h)}$ but usually positive root is expected.

2. Simplify $\frac{3}{(\sqrt{5} - \sqrt{2})}$ by rationalizing the denominator. (3 marks)

Multiply numerator and denominator by conjugate $\sqrt{5} + \sqrt{2}$:

$\frac{3(\sqrt{5} + \sqrt{2})}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})}$ [1]

Denominator simplifies to $5 - 2 = 3$ [1]

Result: $\sqrt{5} + \sqrt{2}$ [1]

3. A rectangle measures 8.5 cm by 5.2 cm, each measured to the nearest 0.1 cm. (3 marks)

Calculate the percentage error in the area.

Maximum length = 8.55, minimum = 8.45; max width = 5.25, min = 5.15 [1]

Maximum area = $8.55 \times 5.25 = 44.8875$, minimum area = $8.45 \times 5.15 = 43.5175$, actual area = 44.2 [1]

Absolute error = $(44.8875 - 43.5175)/2 = 0.685$; percentage error = $(0.685/44.2) \times 100 = 1.55\%$ [1]

Alternative method/answer:

Accept 1.55% or 1.5% (if rounded).

4. y varies partly as x and partly as the square of x . When $x = 2$, $y = 14$; when $x = 3$, $y = 30$. Find y when $x = 4$. (3 marks)

Let $y = ax + bx^2$. Form equations: $2a + 4b = 14$, $3a + 9b = 30$ [1]

Solve to get $a = 1$, $b = 3$ [1]

When $x = 4$, $y = 4 + 3(16) = 52$ [1]

5. The third term of a GP is 12 and the sixth term is 96. Find the first term and the common ratio. (3 marks)

Let first term a , common ratio r . $ar^2 = 12$, $ar^5 = 96$ [1]

Divide: $r^3 = 8 \Rightarrow r = 2$ [1]

Substitute: $a(2)^2 = 12 \Rightarrow a = 3$ [1]

6. Solve the equation $x^2 + 6x + 2 = 0$ by completing the square, leaving your answer in surd form. (3 marks)

Rewrite: $x^2 + 6x = -2$; add $(6/2)^2 = 9$: $x^2 + 6x + 9 = 7$ [1]

Factor: $(x + 3)^2 = 7$ [1]

Take square root: $x = -3 \pm \sqrt{7}$ [1]

7. Expand $(2 - x)^5$ in ascending powers of x up to the term in x^3 . (3 marks)

First term: $\text{binom}{5}{0}2^5 = 32$; second: $\text{binom}{5}{1}2^4(-x) = -80x$ [1]

Third: $\text{binom}{5}{2}2^3(-x)^2 = 80x^2$; fourth: $\text{binom}{5}{3}2^2(-x)^3 = -40x^3$ [1]

Expansion: $32 - 80x + 80x^2 - 40x^3$ [1]

Alternative method/answer:

Accept expansion up to x^3 term only.

- 8.** In the diagram below, O is the centre of the circle. AB is a chord of length 12 cm perpendicular to OX at X. If OX = 8 cm, find the radius of the circle. (3 marks)

Perpendicular from centre bisects chord, so AX = XB = 6 cm [1]

In right triangle OXA, $OA^2 = OX^2 + AX^2 = 8^2 + 6^2 = 100$ [1]

Radius OA = 10 cm [1]

- 9.** Solve the equation $\sin 2\theta = \cos \theta$ for $0^\circ \leq \theta \leq 360^\circ$. (4 marks)

Use identity $\sin 2\theta = 2\sin\theta\cos\theta$: $2\sin\theta\cos\theta = \cos\theta$ [1]

Rearrange: $\cos\theta(2\sin\theta - 1) = 0$ [1]

Case 1: $\cos\theta = 0 \Rightarrow \theta = 90^\circ, 270^\circ$ [1]

Case 2: $\sin\theta = 1/2 \Rightarrow \theta = 30^\circ, 150^\circ$ [1]

Alternative method/answer:

Accept solutions in degrees: $30^\circ, 90^\circ, 150^\circ, 270^\circ$.

- 10.** Given vectors $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$, find the magnitude of $2\mathbf{a} - \mathbf{b}$. (4 marks)

$2\mathbf{a} = \begin{pmatrix} 4 \\ -2 \\ 6 \end{pmatrix}$ [1]

$2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 3 \\ -6 \\ 8 \end{pmatrix}$ [1]

Magnitude = $\sqrt{3^2 + (-6)^2 + 8^2} = \sqrt{9 + 36 + 64}$ [1]

= $\sqrt{109}$ [1]

- 11.** A transformation is represented by matrix $M = \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$. Find the image of the point P(2, -3) under M. (4 marks)

Write point as column vector: $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ [1]

Multiply: $\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ [1]

Compute: $\begin{pmatrix} 2 \times 2 + (-1) \times (-3) \\ 3 \times 2 + 0 \times (-3) \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$ [1]

Image point: (7,6) [1]

- 12.** A bag contains 4 red and 6 blue marbles. Two marbles are drawn without replacement. Find the probability that both are red. (4 marks)

$P(\text{first red}) = (4)/(10) = (2)/(5)$ [1]

After one red, remaining: 3 red, 6 blue, total 9 [1]

$P(\text{second red}) = (3)/(9) = (1)/(3)$ [1]

$P(\text{both red}) = (2)/(5) \times (1)/(3) = (2)/(15)$ [1]

Alternative method/answer:

Accept $(2)/(15)$ or 0.1333.

- 13.** Evaluate $\int_1^2 (3x^2 - 2x + 1) \, dx$. (4 marks)

Integrate: $\int (3x^2 - 2x + 1) \, dx = x^3 - x^2 + x$ [1]

Evaluate at upper limit $x=2$: $8 - 4 + 2 = 6$ [1]

Evaluate at lower limit $x=1$: $1 - 1 + 1 = 1$ [1]

Difference: $6 - 1 = 5$ [1]

- 14.** A rectangular block has dimensions AB = 6 cm, BC = 8 cm, and AE = 10 cm. Find the angle between the diagonal AG and the base ABCD. (4 marks)

Base diagonal AC = $\sqrt{6^2 + 8^2} = 10$ cm [1]

Space diagonal AG = $\sqrt{10^2 + 10^2} = 10\sqrt{2}$ cm [1]

Angle between AG and base: $\cos\theta = (AC)/(AG) = (10)/(10\sqrt{2}) = (1)/(\sqrt{2})$ [1]

$\theta = 45^\circ$ [1]

Alternative method/answer:

Accept $\theta = 45^\circ$ or $\pi/4$ rad.

15. Two points A and B on the Equator have longitudes 30°E and 45°W respectively. Calculate the distance between them along the Equator, taking the Earth's radius as 6370 km. (Use $\pi = 3.142$) (4 marks)

Longitude difference = $30^\circ - (-45^\circ) = 75^\circ$ [1]

Distance = $\frac{\theta}{360} \times 2\pi R$ [1]

Substitute: $\frac{75}{360} \times 2 \times 3.142 \times 6370$ [1]

Distance = 8333.5 km (approx) [1]

Alternative method/answer:

Accept 8333.5 km or 8334 km.

16. Construct the locus of points equidistant from two intersecting lines forming an angle of 60° . Describe the locus. (4 marks)

Identify that locus is the angle bisector [2]

Description: The locus is the set of points equidistant from the two lines, i.e., the angle bisector of the 60° angle. [2]

Alternative method/answer:

Accept: 'The angle bisector of the angle between the two lines'.

SECTION II (50 marks)

17. In a certain country, income tax is calculated as follows:
 First Ksh Sh. 10,000 per month: tax rate 10%.
 Next Ksh Sh. 20,000 per month: tax rate 15%.
 Next Ksh Sh. 30,000 per month: tax rate 20%.
 Above Ksh Sh. 60,000 per month: tax rate 25%.
 A person earns a monthly salary of Ksh Sh. 85,000. They are entitled to a personal relief of Ksh Sh. 1,200 per month. (a) (4 marks) (b) (2 marks) (c) If the person also contributes 5% of their salary to a pension scheme which is tax exempt, find the new net tax payable. (4 marks) (10 marks)

Tax on first 10,000: $0.10 \times 10000 = 1000$ [1]

Tax on next 20,000: $0.15 \times 20000 = 3000$ [1]

Tax on next 30,000: $0.20 \times 30000 = 6000$ [1]

Remaining income: $85000 - 60000 = 25000$, tax: $0.25 \times 25000 = 6250$ [1]

Gross tax = $1000 + 3000 + 6000 + 6250 = 16250$ [1]

Net tax = $16250 - 1200 = 15050$ [1]

Pension contribution: $0.05 \times 85000 = 4250$, taxable income = 80750 [1]

Recalculate tax: first 10k: 1000, next 20k: 3000, next 30k: 6000, remaining 20750: $0.25 \times 20750 = 5187.5$ [1]

Gross tax = $1000 + 3000 + 6000 + 5187.5 = 15187.5$ [1]

New net tax = $15187.5 - 1200 = 13987.5$ [1]

Alternative method/answer:

Accept Sh. 16,250; Sh. 15,050; Sh. 13,987.50 respectively.

18. The table below shows the distribution of marks scored by 100 students in a Mathematics test.
 Marks & Frequency
 10-19 & 4
 20-29 & 12
 30-39 & 20
 40-49 & 30
 50-59 & 18
 60-69 & 10
 70-79 & 6
 (a) (4 marks) (b) Using your ogive, estimate:
 (i) The median mark. (2 marks) (ii) The interquartile range. (2 marks) (c) (2 marks) (10 marks)

Cumulative frequencies: 0, 4, 16, 36, 66, 84, 94, 100 [1]

Plot points at upper boundaries: 9.5, 19.5, 29.5, 39.5, 49.5, 59.5, 69.5, 79.5 and draw smooth curve [1]

Median: 50th student, read from ogive ≈ 46 [2]

Q1: 25th student ≈ 36 , Q3: 75th student ≈ 56 , IQR = 20 [2]

Midpoints: 14.5, 24.5, 34.5, 44.5, 54.5, 64.5, 74.5 [1]

$$\text{Mean} = (14.5 \times 4 + 24.5 \times 12 + 34.5 \times 20 + 44.5 \times 30 + 54.5 \times 18 + 64.5 \times 10 + 74.5 \times 6) / 100 = 45.9 \text{ [1]}$$

$$\text{Variance} = [4(14.5 - 45.9)^2 + 12(24.5 - 45.9)^2 + 20(34.5 - 45.9)^2 + 30(44.5 - 45.9)^2 + 18(54.5 - 45.9)^2 + 10(64.5 - 45.9)^2 + 6(74.5 - 45.9)^2] / 100 = 246.19 \text{ [2]}$$

Alternative method/answer:

Accept median ≈ 46 , IQR ≈ 20 , variance ≈ 246.19 (or 246.2).

- 19.** A triangle with vertices A(1,2), B(3,4) and C(5,1) undergoes two successive transformations. First transformation: Rotation through 90° anticlockwise about the origin, represented by matrix R. Second transformation: Reflection in the line $y = x$, represented by matrix M. (a) Write down the matrices R and M. (2 marks) (b) Find the image of point A after the first transformation. (2 marks) (c) Determine the single transformation matrix equivalent to the combined transformation $M \circ R$ (i.e., first R, then M). (3 marks) (d) (3 marks) (10 marks)

$$R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ [2]}$$

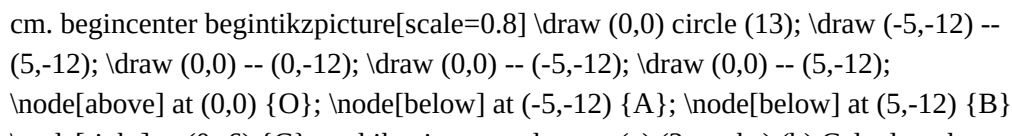
$$\text{A after first: } R \times \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \rightarrow (-2, 1) \text{ [2]}$$

$$\text{Combined } T = M \times R = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ [3]}$$

$$\text{Apply } T \text{ to A: } (1, -2); B: (3, -4); C: (5, -1) \text{ [3]}$$

Alternative method/answer:

Accept coordinates as ordered pairs.

- 20.** In the diagram below, O is the centre of the circle. AB is a chord, and C is a point on the circle such that OC is perpendicular to AB. Given that AB = 24 cm and OC = 5 cm.  (a) (3 marks) (b) Calculate the angle AOB correct to 1 decimal place. (3 marks) (c) Determine the area of the minor segment ACB. (4 marks) (10 marks)

$$AC = 12 \text{ cm (half of AB) [1]}$$

$$\text{Radius } OA = \sqrt{5^2 + 12^2} = 13 \text{ cm [2]}$$

$$\sin(\text{angle } AOC/2) = 12/13, \text{ so angle } AOC/2 = \arcsin(12/13) \approx 67.38^\circ \text{ [1]}$$

$$\text{angle } AOB = 2 \times 67.38 = 134.76^\circ \approx 134.8^\circ \text{ [2]}$$

$$\text{Area of sector} = (134.8/360) \times \pi \times 13^2 \approx 198.8 \text{ cm}^2 \text{ [2]}$$

$$\text{Area of triangle } AOB = 0.5 \times 13 \times 13 \times \sin 134.8^\circ \approx 59.8 \text{ cm}^2 \text{ [1]}$$

$$\text{Segment area} = 198.8 - 59.8 = 139.0 \text{ cm}^2 \text{ [1]}$$

Alternative method/answer:

Accept 139.0 cm^2 or 139 cm^2 .

- 21.** Points P, Q, R have position vectors $\vec{p} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{q} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, $\vec{r} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$. (a) Find the vector $\overrightarrow{PQ}$ and $\overrightarrow{QR}$. (2 marks) (b) Show that P, Q, R are collinear. (3 marks) (c) Find the ratio PQ:QR. (2 marks) (d) Determine the coordinates of point S such that Q is the midpoint of PS. (3 marks) (10 marks)

$$\overrightarrow{PQ} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}, \overrightarrow{QR} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \text{ [2]}$$

Since $\overrightarrow{PQ} = \overrightarrow{QR}$, they are parallel and share point Q, so P, Q, R are collinear. [3]

$$\text{Magnitudes: } |\overrightarrow{PQ}| = 3\sqrt{3}, |\overrightarrow{QR}| = 3\sqrt{3}, \text{ ratio } PQ:QR = 1:1 \text{ [2]}$$

$$\text{Midpoint condition: } \vec{q} = (\vec{p} + \vec{s})/2 \Rightarrow \vec{s} = 2\vec{q} - \vec{p} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix} \text{ [3]}$$

Alternative method/answer:

Accept S(7,8,9).

- 22.** Consider the function $f(x) = 3\sin(2x) + 1$ for $0^\circ \leq x \leq 360^\circ$. (a) (10 marks)
Determine the amplitude, period, and phase shift of f . (3 marks) (b) Sketch the graph of f for the given interval, clearly labeling key points. (4 marks) (c) Solve the equation $f(x) = 2$ for $0^\circ \leq x \leq 360^\circ$. (3 marks)

Amplitude = 3, period = 180° , phase shift = 0 [3]

Graph: correct shape, key points: (0,1), (45,4), (90,1), (135,-2), (180,1), (225,4), (270,1), (315,-2), (360,1) [4]

Solve $3\sin 2x + 1 = 2 \Rightarrow \sin 2x = 1/3$ [1]

$2x = 19.47^\circ, 160.53^\circ, 379.47^\circ, 520.53^\circ$; $x = 9.735^\circ, 80.265^\circ, 189.735^\circ, 260.265^\circ$ [2]

Alternative method/answer:

Accept $x \approx 9.7^\circ, 80.3^\circ, 189.7^\circ, 260.3^\circ$.

- 23.** Two towns A and B lie on the equator. A is at longitude 30°E , and B is at longitude 60°E . Take the radius of the earth as 6370 km. (a) Calculate the distance between A and B along the equator in kilometres. (3 marks) (b) A plane flies from A due north to C at latitude 45°N , then due east to D at longitude 60°E , then due south back to B. Determine the total distance travelled. (7 marks) (10 marks)

Angular difference = 30° , distance = $(30/360) \times 2\pi \times 6370 = 3335.8$ km [3]

A to C: latitude change 45° , distance = $(45/360) \times 2\pi \times 6370 = 5005.8$ km [2]

Radius at $45^\circ\text{N} = 6370 \times \cos 45^\circ = 4504.5$ km [1]

C to D: longitude difference 30° , distance = $(30/360) \times 2\pi \times 4504.5 = 2358.5$ km [2]

D to B: same as A to C = 5005.8 km [1]

Total = $5005.8 + 2358.5 + 5005.8 = 12370.1$ km [1]

Alternative method/answer:

Accept 12370 km or 12370.1 km.

- 24.** A particle moves along a straight line with velocity $v(t) = 3t^2 - 12t + 9$ m/s, where t is time in seconds, $t \geq 0$. (a) (1 mark) (b) (2 marks) (c) (3 marks) (d) (4 marks) (10 marks)

Initial velocity: $v(0) = 9$ m/s [1]

Set $v=0$: $3t^2 - 12t + 9 = 0 \Rightarrow t^2 - 4t + 3 = 0 \Rightarrow (t-1)(t-3) = 0$ [1]

Times: $t=1$ s and $t=3$ s [1]

Displacement = $\int_0^4 v \, dt = [t^3 - 6t^2 + 9t]_0^4 = (64 - 96 + 36) - 0 = 4$ m [3]

Distance: from 0 to 1: $\int_0^1 v \, dt = 4$ m; from 1 to 3: $\int_1^3 -v \, dt = 4$ m; from 3 to 4: $\int_3^4 v \, dt = 4$ m [3]

Total distance = $4 + 4 + 4 = 12$ m [1]