

EDUZEDCO

121/2 MATHEMATICS

Paper 2

Form 4 End Term Examination

Time: 2 ½ Hours

Name:

Adm No:

School:

Class:

Signature:

Date:

INSTRUCTIONS TO CANDIDATES

- (a) Write your name and index number in the spaces provided at the top of this page.
- (b) Write your school name, sign and write the date of the examination in the spaces provided above.
- (c) This paper consists of **Two** sections: **Section I** and **Section II**.
- (d) Answer all questions as instructed in each section.
- (e) Marks may be given for correct working, relevant points, and clear presentation.
- (f) Non-programmable silent electronic calculators may be used where necessary.
- (g) KNEC Mathematical Tables may be used.
- (h) This paper consists of **7 printed pages**.
- (i) Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.
- (j) Candidates should answer the questions in English.

For Examiner's Use Only

SECTION I

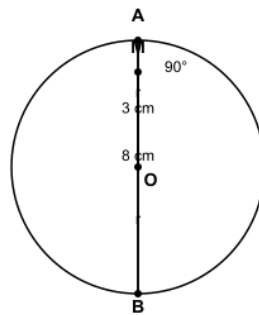
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

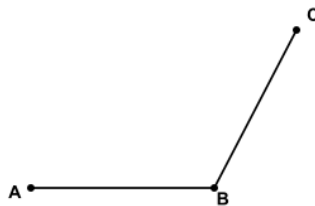
17	18	19	20	21	Total	Grand Total

SECTION I

1. Make x the subject of the formula: $y = \frac{3x+2}{x-4}$.
2. Simplify $\frac{5}{\sqrt{5}-2}$ by rationalising the denominator.
3. The length of a rope is measured as 12.5 cm to the nearest 0.1 cm. Find the percentage error in the measurement.
4. y varies inversely as x . When $x = 3$, $y = 12$. Find y when $x = 9$.
5. The third term of a geometric progression is 20 and the sixth term is 160. Find the common ratio.
6. Express $2x^2 + 8x + 3$ in the form $a(x+h)^2 + k$.
7. Expand $(1+2x)^5$ up to the term in x^3 .
8. In the diagram, O is the centre of the circle, AB is a chord of length 8 cm, and OM is perpendicular to AB and $OM = 3$ cm. Find the radius of the circle.

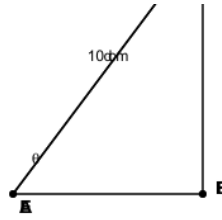


9. Solve the equation $\sin 2x = 0.5$ for $0^\circ \leq x < 360^\circ$.
10. Given vectors $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$, find $|\mathbf{a} - \mathbf{b}|$.



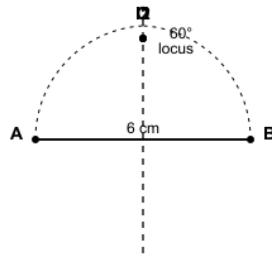
11. The matrix $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$. Find A^{-1} .
12. A bag contains 5 red and 3 blue marbles. Two marbles are drawn without replacement. Find the probability that both are red.
13. Evaluate $\int_1^3 (2x^2 - 3x + 1) dx$.

14. A rectangular box has dimensions $6 \text{ cm} \times 8 \text{ cm} \times 10 \text{ cm}$. Find the angle between the diagonal of the base and the diagonal of the box (space diagonal).



15. Two points $P(30^\circ N, 45^\circ E)$ and $Q(30^\circ N, 15^\circ W)$ lie on the same latitude. Find the distance between them along the parallel of latitude. (Take radius of earth $R = 6400 \text{ km}$)

16. Construct the locus of a point P that is equidistant from two fixed points A and B (distance $AB = 6 \text{ cm}$) and also equidistant from two intersecting lines l_1 and l_2 that intersect at O at an angle of 60° . Describe the locus.



SECTION II

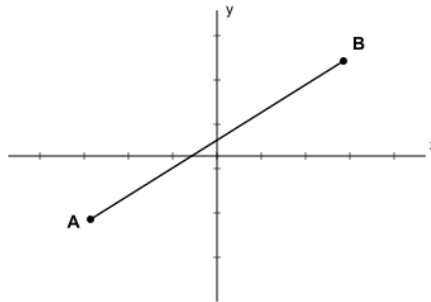
Answer any five questions from this section.

17. The income tax rates for a certain year are as follows: | Monthly Income (Ksh) | Tax Rate (%) |
|-----|-----| 0 – 10,000 | 10 | | 10,001 – 30,000 | 15 | | 30,001 – 60,000 | 20 | | Over 60,000 | 25 | A person earns a monthly salary of Ksh 45,000. They are entitled to a personal relief of Ksh 1,000 per month. Calculate the net tax payable per month. [10 marks]

(a) Calculate the gross tax before relief.

(b) Calculate the net tax after relief.

18. The marks of 50 students in a test are given in the table below. | Marks | Frequency | |-----|-----|
10-19 | 4 | | 20-29 | 8 | | 30-39 | 12 | | 40-49 | 14 | | 50-59 | 8 | | 60-69 | 4 | (a) Draw a cumulative frequency curve (ogive) for the data. (b) Estimate the median mark. (c) Estimate the interquartile range. [10 marks]

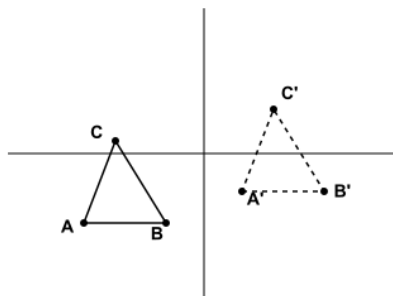


(a) Draw the ogive.

(b) Estimate the median.

(c) Estimate the interquartile range.

19. A transformation T is represented by matrix $M = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ followed by a rotation of 90° anticlockwise about the origin represented by matrix $R = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. (a) Find the single matrix for the combined transformation. (b) Determine the image of the point $(3, -2)$ under the combined transformation. [10 marks]

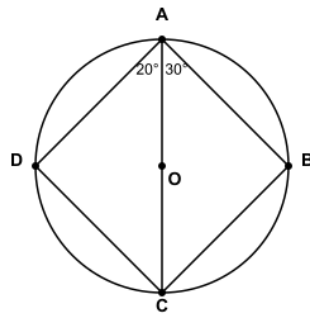


(a) Find the combined matrix.

(b) Find the image of $(3, -2)$.

20. In the diagram, O is the centre of the circle. AB is a diameter. C is a point on the circle such that $\angle BAC = 30^\circ$. D is a point on the circle such that AD is a chord and $\angle CAD = 20^\circ$. Find $\angle BCD$.

[10 marks]



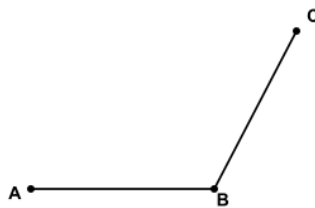
(a) Find $\angle ABC$.

(b) Find $\angle ADC$.

(c) Find $\angle BCD$.

21. Points A , B , and C have position vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}$, and $\mathbf{c} = \begin{pmatrix} 5 \\ 8 \\ 11 \end{pmatrix}$ respectively.

(a) Show that A , B , and C are collinear. (b) Find the ratio $AB : BC$. [10 marks]



(a) Show collinearity.

(b) Find ratio.

121/2 MATHEMATICS MARKING SCHEME

Form 4 Paper 2

SECTION I (50 marks)

1. Make x the subject of the formula: $y = (3x + 2)/(x - 4)$. (2 marks)

Multiply both sides by $(x-4)$: $y(x-4) = 3x+2$ [1]

Expand and collect x terms: $yx - 4y = 3x+2 \rightarrow yx - 3x = 4y+2$ [0.5]

Factor x : $x(y-3) = 4y+2 \rightarrow x = (4y+2)/(y-3)$ [0.5]

Alternative method/answer:

$$x = (2(2y+1))/(y-3)$$

2. Simplify $(5)/(\sqrt{5} - 2)$ by rationalising the denominator. (2 marks)

Multiply numerator and denominator by conjugate $(\sqrt{5}+2)$: $5(\sqrt{5}+2)/((\sqrt{5}-2)(\sqrt{5}+2))$ [1]

Denominator simplifies to $5-4=1$, numerator = $5\sqrt{5}+10$ [1]

Alternative method/answer:

$$5\sqrt{5}+10$$

3. The length of a rope is measured as 12.5 cm to the nearest 0.1 cm. Find the percentage error in the measurement. (3 marks)

Absolute error = $0.5 \times 0.1 = 0.05$ cm [1]

Percentage error = $(0.05/12.5) \times 100\%$ [1]

$$= 0.4\% \text{ [1]}$$

4. y varies inversely as x . When $x = 3$, $y = 12$. Find y when $x = 9$. (3 marks)

$y = k/x$, substitute $x=3$, $y=12$: $12 = k/3 \rightarrow k=36$ [1]

When $x=9$: $y = 36/9$ [1]

$$= 4 \text{ [1]}$$

5. The third term of a geometric progression is 20 and the sixth term is 160. Find the common ratio. (3 marks)

$$ar^2 = 20, ar^5 = 160 \text{ [1]}$$

Divide: $r^3 = 160/20 = 8$ [1]

$$r = 2 \text{ [1]}$$

6. Express $2x^2 + 8x + 3$ in the form $a(x + h)^2 + k$. (3 marks)

Factor 2: $2(x^2+4x)+3$ [1]

Complete square: $x^2+4x = (x+2)^2 - 4$, so $2[(x+2)^2-4]+3 = 2(x+2)^2 -8+3$ [1]

$$= 2(x+2)^2 -5 \text{ [1]}$$

7. Expand $(1 + 2x)^5$ up to the term in x^3 . (4 marks)

Binomial coefficients: 1,5,10,10,... [1]

Term 1: 1, Term in x : $5(2x)=10x$ [1]

Term in x^2 : $10(2x)^2=40x^2$ [1]

Term in x^3 : $10(2x)^3=80x^3$ [1]

Alternative method/answer:

$$1+10x+40x^2+80x^3$$

8. In the diagram, O is the centre of the circle, AB is a chord of length 8 cm, and OM is perpendicular to AB and $OM = 3$ cm. Find the radius of the circle. (4 marks)

$AM = MB = 4$ cm (perpendicular from centre bisects chord) [1]

In right triangle OMA: $OA^2 = OM^2 + AM^2 = 3^2 + 4^2 = 25$ [2]
 $OA = 5$ cm [1]

9. Solve the equation $\sin 2x = 0.5$ for $0^\circ \leq x \leq 360^\circ$. (4 marks)

$$\sin 2x = 0.5 \rightarrow 2x = 30^\circ, 150^\circ, 390^\circ, 510^\circ \text{ (reference angles) [2]}$$

$$x = 15^\circ, 75^\circ, 195^\circ, 255^\circ$$
 [2]

Alternative method/answer:

$$x = 15^\circ, 75^\circ, 195^\circ, 255^\circ$$

10. Given vectors $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix}$, find $|\mathbf{a} - \mathbf{b}|$. (4 marks)

$$\mathbf{a} - \mathbf{b} = (2-0, -1-4, 3-(-2)) = (2, -5, 5)$$
 [2]

$$|\mathbf{a} - \mathbf{b}| = \sqrt{2^2 + (-5)^2 + 5^2} = \sqrt{4+25+25} = \sqrt{54}$$
 [1]

$$= 3\sqrt{6}$$
 [1]

Alternative method/answer:

$$\sqrt{54}$$

11. The matrix $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$. Find A^{-1} . (4 marks)

$$\det(A) = (2)(3) - (-1)(1) = 6+1 = 7$$
 [1]

$$A^{-1} = (1/7) \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$
 [2]

$$= \begin{pmatrix} 3/7 & 1/7 \\ 1/7 & 2/7 \end{pmatrix}$$
 [1]

12. A bag contains 5 red and 3 blue marbles. Two marbles are drawn without replacement. Find the probability that both are red. (4 marks)

$$P(\text{first red}) = 5/8$$
 [1]

$$P(\text{second red} | \text{first red}) = 4/7$$
 [1]

$$P(\text{both red}) = (5/8) \times (4/7) = 20/56$$
 [1]

$$= 5/14$$
 [1]

Alternative method/answer:

$$5/14$$

13. Evaluate $\int_1^3 (2x^2 - 3x + 1) dx$. (4 marks)

$$\int (2x^2 - 3x + 1) dx = (2/3)x^3 - (3/2)x^2 + x$$
 [1]

$$\text{At } x=3: (2/3) \times 27 - (3/2) \times 9 + 3 = 18 - 13.5 + 3 = 7.5$$
 [1]

$$\text{At } x=1: (2/3) \times 1 - (3/2) \times 1 + 1 = 2/3 - 1.5 + 1 = 0.1667$$
 [1]

$$\text{Difference} = 7.5 - 0.1667 = 7.3333 = 22/3$$
 [1]

Alternative method/answer:

$$22/3$$

14. A rectangular box has dimensions 6 cm x 8 cm x 10 cm. Find the angle between the diagonal of the base and the diagonal of the box (space diagonal). (4 marks)

$$\text{Base diagonal } AC = \sqrt{6^2 + 8^2} = 10$$
 cm [1]

$$\text{Space diagonal } AG = \sqrt{6^2 + 8^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$$
 cm [1]

$$\cos \theta = AC/AG = 10/(10\sqrt{2}) = 1/\sqrt{2}$$
 [1]

$$\theta = 45^\circ$$
 [1]

15. Two points P(30° N, 45° E) and Q(30° N, 15° W) lie on the same latitude. Find the distance between them along the parallel of latitude. (Take radius of earth $R = 6400$ km) (4 marks)

$$\text{Longitude difference} = 45^\circ - (-15^\circ) = 60^\circ$$
 [1]

$$\text{Distance} = (60/360) \times 2\pi R \cos 30^\circ = (1/6) \times 2\pi \times 6400 \times (\sqrt{3}/2)$$
 [2]

$$= (6400\pi\sqrt{3})/6 = (3200\pi\sqrt{3})/3$$
 km [1]

Alternative method/answer:

$$(3200\pi\sqrt{3})/3 \text{ km}$$

- 16.** Construct the locus of a point P that is equidistant from two fixed points A and B (distance AB = 6 cm) and also equidistant from two intersecting lines l_1 and l_2 that intersect at O at an angle of 60° . Describe the locus. (4 marks)

Locus equidistant from A and B: perpendicular bisector of AB (1 mark for correct line) [1]

Locus equidistant from l_1 and l_2 : angle bisectors of the 60° angle (1 mark for correct lines) [1]

Intersection points: two points where perpendicular bisector meets each angle bisector (1 mark for identifying both) [1]

Description: two points (1 mark for clear description) [1]

Alternative method/answer:

Two points: intersection of perpendicular bisector of AB and the two angle bisectors of l_1 and l_2 .

SECTION II (50 marks)

- 17.** The income tax rates for a certain year are as follows: | Monthly Income (Ksh) | Tax Rate (%) | |-----|-----| | 0 – 10,000 | 10 | | 10,001 – 30,000 | 15 | | 30,001 – 60,000 | 20 | | Over 60,000 | 25 | A person earns a monthly salary of Ksh 45,000. They are entitled to a personal relief of Ksh 1,000 per month. Calculate the net tax payable per month. (10 marks)

First 10,000: 10% of 10,000 = 1,000 [2]

Next 20,000 (10,001-30,000): 15% of 20,000 = 3,000 [2]

Remaining 15,000 (30,001-45,000): 20% of 15,000 = 3,000 [2]

Gross tax = 1,000 + 3,000 + 3,000 = 7,000 [2]

Net tax = 7,000 - 1,000 = 6,000 [2]

- 18.** The marks of 50 students in a test are given in the table below. | Marks | Frequency | |-----|-----| | 10-19 | 4 | | 20-29 | 8 | | 30-39 | 12 | | 40-49 | 14 | | 50-59 | 8 | | 60-69 | 4 | (a) Draw a cumulative frequency curve (ogive) for the data. (b) Estimate the median mark. (c) Estimate the interquartile range. (10 marks)

Cumulative frequencies: 4,12,24,38,46,50 (2 marks for correct table or points) [2]

Ogive drawn correctly (2 marks for correct axes and smooth curve) [2]

Median from ogive at cumulative 25: approximately 40 marks (2 marks for reading) [2]

Q1 at cumulative 12.5: approximately 29 marks (1 mark) [1]

Q3 at cumulative 37.5: approximately 48 marks (1 mark) [1]

IQR = 48 - 29 = 19 (2 marks) [2]

Alternative method/answer:

Accept values within ± 2 of given estimates.

- 19.** A transformation T is represented by matrix $M = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$ followed by a rotation of 90° anticlockwise about the origin represented by matrix $R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. (a) Find the single matrix for the combined transformation. (b) Determine the image of the point (3, -2) under the combined transformation. (10 marks)

Combined matrix = $R \times M$ (order matters) [2]

$R \times M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 \cdot 2 + (-1) \cdot 0 & 0 \cdot 0 + (-1) \cdot (-1) \\ 1 \cdot 2 + 0 \cdot 0 & 1 \cdot 0 + 0 \cdot (-1) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$ [4]

Image = $\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \cdot 3 + 1 \cdot (-2) \\ 2 \cdot 3 + 0 \cdot (-2) \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$ [4]

Alternative method/answer:

Combined matrix: $\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$, Image: (-2,6)

20. In the diagram, O is the centre of the circle. AB is a diameter. C is a point on the circle such that angle BAC = 30° . D is a point on the circle such that AD is a chord and angle CAD = 20° . Find angle BCD. (10 marks)

$$\angle ACB = 90^\circ \text{ (angle in semicircle) [2]}$$

$$\text{In triangle ABC: } \angle ABC = 180^\circ - 90^\circ - 30^\circ = 60^\circ \text{ [2]}$$

$$\angle ADC = \angle ABC = 60^\circ \text{ (angles subtended by same chord AC) [2]}$$

$$\text{In triangle ACD: } \angle ACD = 180^\circ - (30^\circ + 20^\circ) - 60^\circ = 70^\circ \text{ [2]}$$

$$\angle BCD = \angle ACB - \angle ACD = 90^\circ - 70^\circ = 20^\circ \text{ [2]}$$

Alternative method/answer:

$$20^\circ$$

21. Points A, B, and C have position vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}$, and $\mathbf{c} = \begin{pmatrix} 5 \\ 8 \\ 11 \end{pmatrix}$ respectively. (a) Show that A, B, and C are collinear. (b) Find the ratio AB:BC. (10 marks)

$$\text{Vector AB} = \mathbf{b} - \mathbf{a} = (2, 3, 4) \text{ [2]}$$

$$\text{Vector BC} = \mathbf{c} - \mathbf{b} = (2, 3, 4) \text{ [2]}$$

$$\text{AB} = \text{BC, so vectors are parallel and share point B} \rightarrow \text{collinear [2]}$$

$$\text{Ratio AB:BC} = |\mathbf{AB}| : |\mathbf{BC}| = 1:1 \text{ (since vectors equal magnitude) [4]}$$

Alternative method/answer:

$$\text{AB:BC} = 1:1$$