

EDUZEDCO
121/2 MATHEMATICS
Paper 2
Form 4 End Term Examination

Time: 2 ½ Hours

Name: Adm No:
 School: Class:
 Signature: Date:

INSTRUCTIONS TO CANDIDATES

- (a) Write your name and index number in the spaces provided at the top of this page.
- (b) Write your school name, sign and write the date of the examination in the spaces provided above.
- (c) This paper consists of **Two** sections: **Section I** and **Section II**.
- (d) Answer **ALL** questions in Section I and any **five questions** from Section II.
- (e) Marks may be given for correct working, relevant points, and clear presentation.
- (f) Non-programmable silent electronic calculators may be used where necessary.
- (g) KNEC Mathematical Tables may be used.
- (h) This paper consists of **8 printed pages**.
- (i) Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.
- (j) Candidates should answer the questions in English.

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

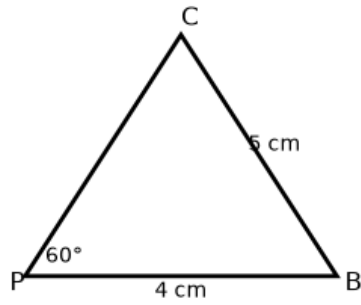
SECTION II

17	18	19	20	21	22	23	24	Total	Grand Total

SECTION I (50 Marks)

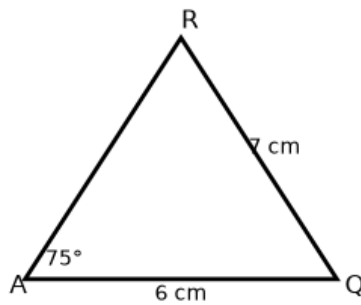
Answer **all** questions in this section in the spaces provided.

1. Make x the subject of the formula $y = \frac{(3x - 2)}{(5)}$. (3 mks)
2. Simplify $\frac{3}{(\sqrt{5} - \sqrt{2})}$ in the form $a\sqrt{5} + b\sqrt{2}$, where a and b are integers. (3 mks)
3. The length of a rectangle is 12.5 cm and its width is 8.2 cm, each measured to the nearest 0.1 cm. Calculate the percentage error in the area. (3 mks)
4. P varies directly as Q and inversely as the square of R . When $Q=8$ and $R=2$, $P=10$. Find P when $Q=18$ and $R=3$. (3 mks)
5. The third term of a geometric progression is 24 and the sixth term is 192. Determine the common ratio and the first term. (3 mks)
6. Solve the equation $x^2 + 6x + 2 = 0$ by completing the square, giving your answer in surd form. (3 mks)
7. Expand $(1 - 2x)^5$ up to the term in x^3 . Use your expansion to evaluate $(0.98)^5$ correct to 4 decimal places. (3 mks)
8. In the figure below, O is the centre of the circle. AB is a chord of length 10 cm and OX is perpendicular to AB meeting AB at X . If $OX = 4$ cm, find the radius of the circle. (3 mks)

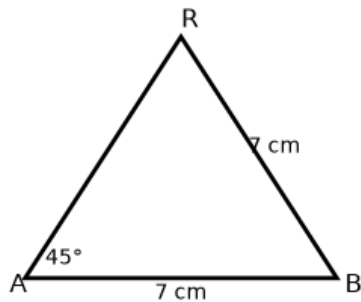


9. Solve the equation $\sin 2\theta = \frac{1}{2}$ for $0^\circ \leq \theta < 360^\circ$. (3 mks)

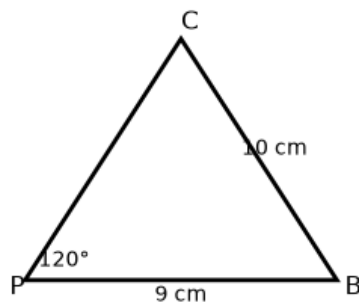
10. Given vectors $\mathbf{p} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$, find $|\mathbf{p} + 2\mathbf{q}|$. (3 mks)



11. The transformation T is represented by the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Find the image of the point $(2, -1)$ under T . (3 mks)



12. A bag contains 5 red and 3 blue pens. Two pens are drawn at random without replacement. Find the probability that they are of different colours. (4 mks)
13. Evaluate $\int_0^2 (3x^2 - 2x + 1) \, dx$. (4 mks)
14. A right pyramid has a square base of side 6 cm and a slant height of 5 cm. Calculate the angle between a slant edge and the base plane. (4 mks)
15. Two points A and B on the equator have longitudes 30°E and 40°E respectively. Calculate the distance between them along the equator, taking the Earth's radius as 6370 km. (Give your answer to the nearest km.) (4 mks)
16. On the grid provided, construct the locus of points equidistant from two fixed points A and B and also equidistant from two intersecting lines L_1 and L_2 . Clearly indicate the intersection points. (4 mks)



SECTION II (50 Marks)

Answer any **five** questions from this section in the spaces provided.

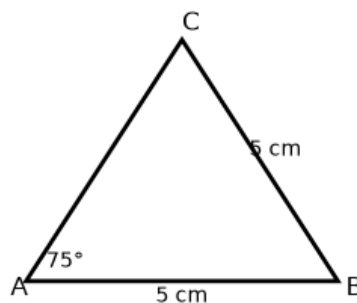
17. In a certain country, income tax is charged on annual income as follows: first 120,000 units tax-free; next 100,000 units at 10%; next 150,000 units at 15%; remaining income at 20%. A person earns a monthly salary of 25,000 units. Calculate (a) the annual income, (b) the tax payable per year, (c) the net monthly income after tax. (10 mks)

Calculate the annual income.

Calculate the tax payable per year.

Calculate the net monthly income after tax.

18. The marks of 80 students in a mathematics test are given in the table below. (10 mks)
- | Mark | Frequency |
|-------|-----------|
| 10-19 | 4 |
| 20-29 | 8 |
| 30-39 | 12 |
| 40-49 | 20 |
| 50-59 | 18 |
| 60-69 | 10 |
| 70-79 | 6 |
| 80-89 | 2 |
- (a) Construct a cumulative frequency table.
(b) Draw an ogive. (c) Estimate the median mark.

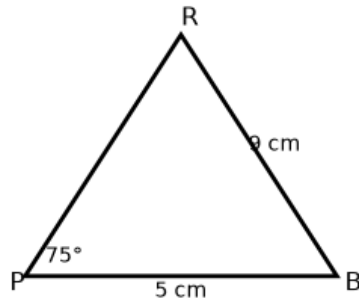


Construct a cumulative frequency table.

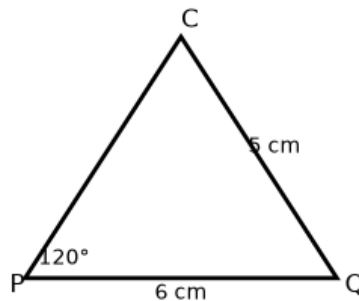
Draw an ogive.

Estimate the median mark.

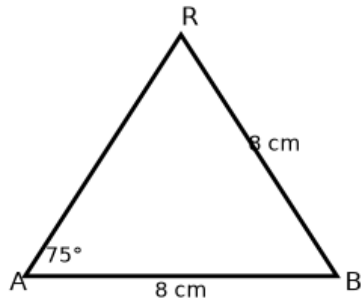
19. A triangle with vertices $A(1,0)$, $B(2,1)$, $C(1,2)$ is transformed by matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ followed by a translation $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$. Find the coordinates of the image of the triangle. (10 mks)



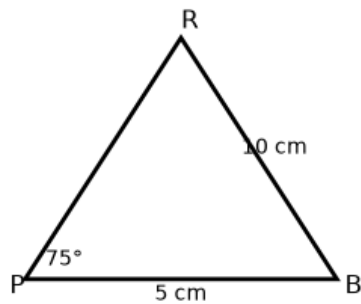
20. In the figure below, O is the centre of the circle. AB is a diameter, C is a point on the circumference such that $\angle BAC = 30^\circ$. D is a point on the minor arc AC . Find $\angle ADC$. (10 mks)



21. Points A , B , C have position vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 5 \\ 6 \\ 11 \end{pmatrix}$ respectively. Show that A , B , C are collinear and find the ratio $AB:BC$. (10 mks)



22. Sketch the graph of $y = 2\sin(3x + 60^\circ)$ for $0^\circ \leq x \leq 180^\circ$. Indicate the amplitude, period, and phase shift. (10 mks)

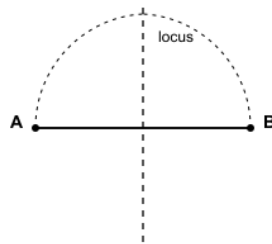


Sketch the graph.

Indicate the amplitude, period, and phase shift.

23. A ship sails from point P at $(10^\circ\text{N}, 20^\circ\text{E})$ to point Q at $(10^\circ\text{N}, 30^\circ\text{E})$ along a parallel of latitude. Calculate the distance sailed in nautical miles. (Take radius of Earth = 3440 nautical miles.) (10 mks)

24. A particle moves along a straight line with velocity $v(t) = 3t^2 - 6t + 2$ m/s, (10 mks)
where t is time in seconds. If the particle starts from the origin, find (a) its
displacement after 4 seconds, (b) the total distance travelled in the first 4
seconds.



Find its displacement after 4 seconds.

Find the total distance travelled in the first 4 seconds.

121/2 MATHEMATICS MARKING SCHEME

Form 4 Paper 2

SECTION I (50 marks)

1. Make x the subject of the formula $y = (3x - 2)/(5)$. (3 marks)

Multiply both sides by 5: $5y = 3x - 2$ [1]

Add 2: $5y + 2 = 3x$ [1]

Divide by 3: $x = (5y + 2)/(3)$ [1]

Alternative method/answer:

Accept $x = (5y+2)/(3)$

2. Simplify $(3)/(\sqrt{5} - \sqrt{2})$ in the form $a\sqrt{5} + b\sqrt{2}$, where a and b are integers. (3 marks)

Multiply numerator and denominator by conjugate: $(3(\sqrt{5}+\sqrt{2}))/((\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}))$ [1]

Simplify denominator: $(\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3$ [1]

Simplify: $(3(\sqrt{5}+\sqrt{2}))/3 = \sqrt{5} + \sqrt{2}$ [1]

Alternative method/answer:

Accept $\sqrt{5}+\sqrt{2}$

3. The length of a rectangle is 12.5 cm and its width is 8.2 cm, each measured to the nearest 0.1 cm. Calculate the percentage error in the area. (3 marks)

Maximum length = 12.55 cm, minimum length = 12.45 cm; maximum width = 8.25 cm, minimum width = 8.15 cm [1]

Maximum area = $12.55 \times 8.25 = 103.5375 \text{ cm}^2$; minimum area = $12.45 \times 8.15 = 101.4675 \text{ cm}^2$; actual area = $12.5 \times 8.2 = 102.5 \text{ cm}^2$ [1]

Absolute error = $(103.5375 - 101.4675)/2 = 1.035 \text{ cm}^2$; percentage error = $(1.035/102.5) \times 100\% = 1.01\%$ (or 1.0%) [1]

Alternative method/answer:

Accept percentage error = $(0.5(0.1/12.5 + 0.1/8.2)) \times 100\% = 1.01\%$

4. P varies directly as Q and inversely as the square of R . When $Q = 8$ and $R = 2$, $P = 10$. Find P when $Q = 18$ and $R = 3$. (3 marks)

Write variation equation: $P = k(Q)/(R^2)$ [1]

Find k : $10 = k(8)/(2^2) \Rightarrow 10 = 2k \Rightarrow k = 5$ [1]

Find P : $P = 5 \times (18)/(3^2) = 5 \times (18)/(9) = 5 \times 2 = 10$ [1]

5. The third term of a geometric progression is 24 and the sixth term is 192. Determine the common ratio and the first term. (3 marks)

Let first term = a , common ratio = r . Then $ar^2 = 24$, $ar^5 = 192$ [1]

Divide: $(ar^5)/(ar^2) = r^3 = (192)/(24) = 8 \Rightarrow r = 2$ [1]

Substitute: $a(2)^2 = 24 \Rightarrow 4a = 24 \Rightarrow a = 6$ [1]

6. Solve the equation $x^2 + 6x + 2 = 0$ by completing the square, giving your answer in surd form. (3 marks)

Rewrite: $x^2 + 6x = -2$; add $(6/2)^2 = 9$: $x^2 + 6x + 9 = 7$ [1]

Factor: $(x+3)^2 = 7$ [1]

Take square root: $x+3 = \pm\sqrt{7} \Rightarrow x = -3 \pm \sqrt{7}$ [1]

7. Expand $(1 - 2x)^5$ up to the term in x^3 . Use your expansion to evaluate $(0.98)^5$ correct to 4 decimal places. (3 marks)

Expand: $(1-2x)^5 = 1 + 5(-2x) + 10(-2x)^2 + 10(-2x)^3 + \dots = 1 - 10x + 40x^2 - 80x^3$ [1]

Set $x = 0.01$ because $0.98 = 1 - 2(0.01)$ [1]

Substitute: $(0.98)^5 \approx 1 - 10(0.01) + 40(0.0001) - 80(0.000001) = 1 - 0.1 + 0.004 - 0.00008 = 0.90392$;
correct to 4 d.p. = 0.9039 [1]

Alternative method/answer:

Accept 0.9039

- 8.** In the figure below, O is the centre of the circle. AB is a chord of length 10 cm and OX is perpendicular to AB meeting AB at X. If OX = 4 cm, find the radius of the circle. (3 marks)

Since OX $\perp$ AB, X is midpoint: AX = 5 cm [1]

In right triangle OXA: $OA^2 = OX^2 + AX^2 = 4^2 + 5^2 = 16 + 25 = 41$ [1]

Radius OA = $\sqrt{41}$ cm [1]

- 9.** Solve the equation $\sin 2\theta = \frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$. (3 marks)

General solution: $\sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = 30^\circ + 360^\circ n$ or $2\theta = 150^\circ + 360^\circ n$ [1]

Divide by 2: $\theta = 15^\circ + 180^\circ n$ or $\theta = 75^\circ + 180^\circ n$ [1]

For $0^\circ \leq \theta \leq 360^\circ$: $\theta = 15^\circ, 75^\circ, 195^\circ, 255^\circ$ [1]

Alternative method/answer:

Accept answers in degrees: $15^\circ, 75^\circ, 195^\circ, 255^\circ$

- 10.** Given vectors $\mathbf{p} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$, find $|\mathbf{p} + 2\mathbf{q}|$. (3 marks)

Compute $2\mathbf{q} = \begin{pmatrix} 8 \\ 0 \\ -4 \end{pmatrix}$ [1]

Add: $\mathbf{p} + 2\mathbf{q} = \begin{pmatrix} 2+8 \\ -1+0 \\ 3+(-4) \end{pmatrix} = \begin{pmatrix} 10 \\ -1 \\ -1 \end{pmatrix}$ [1]

Magnitude: $|\mathbf{p} + 2\mathbf{q}| = \sqrt{10^2 + (-1)^2 + (-1)^2} = \sqrt{100+1+1} = \sqrt{102}$ [1]

- 11.** The transformation T is represented by the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Find the image of the point (2, -1) under T. (3 marks)

Write point as column vector: $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ [1]

Multiply: $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1(2)+2(-1) \\ 3(2)+4(-1) \end{pmatrix} = \begin{pmatrix} 2-2 \\ 6-4 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ [1]

Image point: (0, 2) [1]

- 12.** A bag contains 5 red and 3 blue pens. Two pens are drawn at random without replacement. Find the probability that they are of different colours. (4 marks)

Total pens = 8. Probability first red then blue: $\frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$ [1]

Probability first blue then red: $\frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$ [1]

Add: $\frac{15}{56} + \frac{15}{56} = \frac{30}{56}$ [1]

Simplify: $\frac{15}{28}$ [1]

Alternative method/answer:

Accept $\frac{15}{28}$

- 13.** Evaluate $\int_0^2 (3x^2 - 2x + 1) \, dx$. (4 marks)

Integrate: $\int (3x^2 - 2x + 1) \, dx = x^3 - x^2 + x$ [1]

Substitute upper limit: $2^3 - 2^2 + 2 = 8 - 4 + 2 = 6$ [1]

Substitute lower limit: $0^3 - 0^2 + 0 = 0$ [1]

Subtract: $6 - 0 = 6$ [1]

- 14.** A right pyramid has a square base of side 6 cm and a slant height of 5 cm. (4 marks)

Calculate the angle between a slant edge and the base plane.

Base diagonal = $6\sqrt{2}$ cm; half diagonal = $3\sqrt{2}$ cm [1]

Slant edge = $\sqrt{5^2 + (3\sqrt{2})^2} = \sqrt{25 + 18} = \sqrt{43}$ cm (or use slant height to find edge: edge = $\sqrt{5^2 + 3^2} = \sqrt{34}$)? Actually careful: slant height is face height, not edge. Use right triangle from apex to base center: height = $\sqrt{5^2 - 3^2} = 4$ cm; then edge = $\sqrt{4^2 + (3\sqrt{2})^2} = \sqrt{16+18} = \sqrt{34}$ cm [1]

Angle between edge and base: $\cos \theta = (\text{half diagonal})/(\text{edge}) = (3\sqrt{2})/(\sqrt{34})$ [1]

$\theta = \cos^{-1}((3\sqrt{2})/(\sqrt{34})) \approx \cos^{-1}(0.7276) \approx 43.3^\circ$ [1]

Alternative method/answer:

Accept $\theta = \tan^{-1}((4)/(3\sqrt{2})) \approx 43.3^\circ$

- 15.** Two points A and B on the equator have longitudes 30°E and 40°E respectively. Calculate the distance between them along the equator, taking the Earth's radius as 6370 km. (Give your answer to the nearest km.) (4 marks)

Difference in longitude = $40^\circ - 30^\circ = 10^\circ$ [1]

Distance along equator = $(10)/(360) \times 2\pi \times 6370$ [1]

Compute: $(1)/(36) \times 2 \times 3.1416 \times 6370 \approx 1111.9$ km [1]

To nearest km: 1112 km [1]

Alternative method/answer:

Accept 1112 km

- 16.** On the grid provided, construct the locus of points equidistant from two fixed points A and B and also equidistant from two intersecting lines l_1 and l_2 . Clearly indicate the intersection points. (4 marks)

Draw perpendicular bisector of AB (1 mark for correct construction) [1]

Draw angle bisectors of lines l_1 and l_2 (1 mark for correct construction) [1]

Identify intersection points of perpendicular bisector and angle bisectors (1 mark for correct identification) [1]

Clearly indicate the intersection points (1 mark for clear indication) [1]

SECTION II (50 marks)

- 17.** In a certain country, income tax is charged on annual income as follows: first 120,000 units tax-free; next 100,000 units at 10%; next 150,000 units at 15%; remaining income at 20%. A person earns a monthly salary of 25,000 units. Calculate (a) the annual income, (b) the tax payable per year, (c) the net monthly income after tax. (10 marks)

Annual income = $25,000 \times 12 = 300,000$ units [2]

Taxable income: first 120,000 tax-free; next 100,000 at 10%: tax = 10,000; next 80,000 (since $300,000 - 220,000 = 80,000$) at 15%: tax = 12,000; total tax = $10,000 + 12,000 = 22,000$ units [4]

Net annual income = $300,000 - 22,000 = 278,000$ units; net monthly income = $278,000/12 \approx 23,166.67$ units [4]

Alternative method/answer:

Accept 23,166.67 or 23,167 units

- 18.** The marks of 80 students in a mathematics test are given in the table below. (10 marks)

Mark	Frequency
10-19	4
20-29	8
30-39	12
40-49	20
50-59	18
60-69	10
70-79	6
80-89	2

(a) Construct a cumulative frequency table. (b) Draw an ogive. (c) Estimate the median mark.

Cumulative frequency table: (10-19:4), (20-29:12), (30-39:24), (40-49:44), (50-59:62), (60-69:72), (70-79:78), (80-89:80) [3]

Ogive: plot points at upper boundaries (19.5,4), (29.5,12), (39.5,24), (49.5,44), (59.5,62), (69.5,72), (79.5,78), (89.5,80); join with smooth curve [4]

Median: 40th value; from ogive, median ≈ 49.5 (or read from graph, accept 49-50) [3]

Alternative method/answer:

Accept median between 49 and 50

- 19.** A triangle with vertices A(1,0), B(2,1), C(1,2) is transformed by matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ followed by a translation $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$. Find the coordinates of the image of the triangle. (10 marks)

Apply M to each vertex: A: (1,0) $\rightarrow$ (2,1); B: (2,1) $\rightarrow$ (5,4); C: (1,2) $\rightarrow$ (4,5) [4]

Apply translation: add (-2,1) to each: A' = (0,2); B' = (3,5); C' = (2,6) [4]

Coordinates: A'(0,2), B'(3,5), C'(2,6) [2]

- 20.** In the figure below, O is the centre of the circle. AB is a diameter, C is a point on the circumference such that angle BAC = 30° . D is a point on the minor arc AC. Find angle ADC. (10 marks)

Angle ACB = 90° (angle in semicircle) [2]

In triangle ABC, angle ABC = $180^\circ - 90^\circ - 30^\circ = 60^\circ$ [2]

Angle ADC = angle ABC (angles subtended by same arc AC) [4]

Therefore angle ADC = 60° [2]

Alternative method/answer:

Accept 60°

- 21.** Points A, B, C have position vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 5 \\ 6 \\ 11 \end{pmatrix}$ respectively. Show that A, B, C are collinear and find the ratio AB:BC. (10 marks)

Find vector AB = $\mathbf{b} - \mathbf{a} = (2, 2, 4)$ [2]

Find vector BC = $\mathbf{c} - \mathbf{b} = (2, 2, 4)$ [2]

Since AB = BC, they are parallel and share point B, so collinear [3]

Ratio AB:BC = 1:1 (since AB = BC) [3]

Alternative method/answer:

Accept ratio 1:1

- 22.** Sketch the graph of $y = 2\sin(3x + 60^\circ)$ for $0^\circ \leq x \leq 180^\circ$. Indicate the amplitude, period, and phase shift. (10 marks)

Amplitude = 2 [2]

Period = $360^\circ/3 = 120^\circ$ [2]

Phase shift = $-60^\circ/3 = -20^\circ$ (or 20° to the left) [2]

Sketch: correct shape, key points at $x = -20^\circ, 10^\circ, 40^\circ, 70^\circ, 100^\circ$ (within domain 0° to 180° , shift accordingly) [4]

Alternative method/answer:

Accept phase shift as 20° left

- 23.** A ship sails from point P at (10°N , 20°E) to point Q at (10°N , 30°E) along a parallel of latitude. Calculate the distance sailed in nautical miles. (Take radius of Earth = 3440 nautical miles.) (10 marks)

Difference in longitude = $30^\circ - 20^\circ = 10^\circ$ [2]

Distance along parallel = $(10/360) \times 2\pi \times R \times \cos(\text{latitude}) = (10/360) \times 2\pi \times 3440 \times \cos(10^\circ)$ [4]

Compute: $(1/36) \times 2 \times 3.1416 \times 3440 \times 0.9848 \approx 591.5$ nautical miles [4]

Alternative method/answer:

Accept 592 nautical miles

24. A particle moves along a straight line with velocity $v(t) = 3t^2 - 6t + 2$ m/s, where t is time in seconds. If the particle starts from the origin, find (a) its displacement after 4 seconds, (b) the total distance travelled in the first 4 seconds. (10 marks)

Displacement $s(t) = \int v \, dt = t^3 - 3t^2 + 2t + C$; since starts at origin, $C=0$ [2]

$s(4) = 64 - 48 + 8 = 24$ m [2]

Find when $v=0$: $3t^2 - 6t + 2 = 0 \rightarrow t = (6 \pm \sqrt{36-24})/6 = (6 \pm \sqrt{12})/6 = 1 \pm \sqrt{3}/3 \approx 0.423, 1.577$ [2]

Distance = $|s(0.423)-0| + |s(1.577)-s(0.423)| + |s(4)-s(1.577)|$; compute $s(0.423)=0.423^3$

$-3(0.423)^2+2(0.423)=0.0757-0.536+0.846=0.3857$; $s(1.577)=3.92-7.46+3.154=-0.386$; $s(4)=24$ [2]

Total distance = $0.3857 + |-0.386 - 0.3857| + |24 - (-0.386)| = 0.3857 + 0.7717 + 24.386 = 25.5434$ m ≈ 25.54 m [2]

Alternative method/answer:

Accept 25.54 m