

EDUZEDCO
121/2 MATHEMATICS
Paper 2
Form 4 End Term Examination

Time: 2 ½ Hours

Name: Adm No:
 School: Class:
 Signature: Date:

INSTRUCTIONS TO CANDIDATES

- (a) Write your name and index number in the spaces provided at the top of this page.
- (b) Write your school name, sign and write the date of the examination in the spaces provided above.
- (c) This paper consists of **Two** sections: **Section I** and **Section II**.
- (d) Answer **ALL** questions in Section I and any **five questions** from Section II.
- (e) Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question.
- (f) Marks may be given for correct working even if the answer is wrong.
- (g) Non-Programmable silent electronic calculators and KNEC Mathematical Tables may be used.
- (h) This paper consists of **8 printed pages**.
- (i) Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.
- (j) Candidates should answer the questions in English.

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total	Grand Total

SECTION I (50 Marks)

Answer **all** questions in this section in the spaces provided.

1. Make p the subject of the formula: (3 mks)

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

2. Simplify $\sqrt{75} + \sqrt{12} - \sqrt{48}$. (3 mks)

3. A rectangular field measures 50.5 m by 30.2 m , each correct to the nearest 0.1 m . Calculate the percentage error in the area. (3 mks)

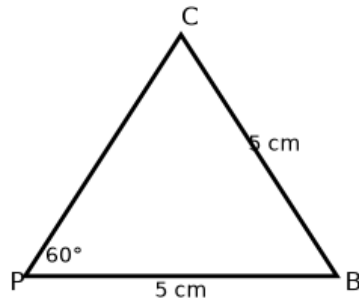
4. y varies directly as the square of x and inversely as z . When $x=2$ and $z=3$, $y=8$. Find y when $x=3$ and $z=4$. (3 mks)

5. The third term of a geometric progression is 12 and the sixth term is 96 . Find the common ratio and the first term. (3 mks)

6. Solve the equation $x^2 - 6x + 2 = 0$ by completing the square. (3 mks)

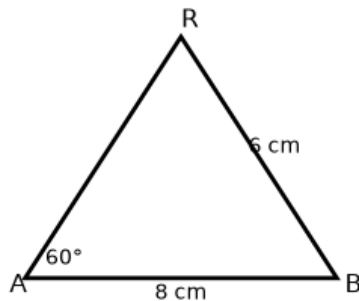
7. Expand $(2-x)^5$ up to the term in x^3 . (3 mks)

8. In the diagram, O is the centre of the circle. PQ is a chord perpendicular to OR where R is the midpoint of PQ . If $PQ=8$ cm and $OR=3$ cm, find the radius of the circle. (3 mks)

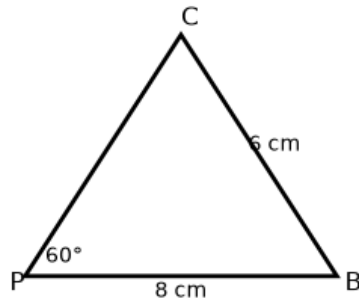


9. Solve the equation $\sin 2\theta = \frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$. (4 mks)

10. Given vectors $\vec{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, find $|\vec{a} - 2\vec{b}|$. (3 mks)



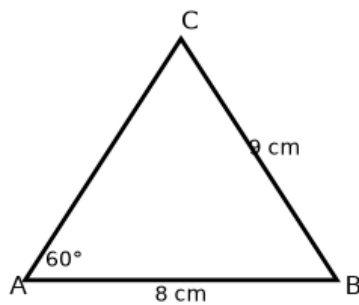
11. A transformation T is represented by the matrix $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$. Find the image of the point $(2, -1)$ under T . (4 mks)



12. A bag contains 3 red, 4 white, and 5 blue balls. Two balls are drawn at random without replacement. Find the probability that both are of the same colour. (4 mks)

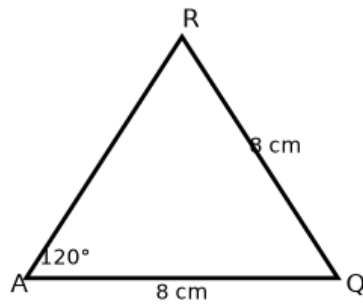
13. Evaluate $\int_0^1 (3x^2 - 2x + 1) dx$. (4 mks)

14. A rectangular block has dimensions $AB=5\text{ cm}$, $BC=4\text{ cm}$, and $AE=3\text{ cm}$. Find the angle between the diagonal AG and the base $ABCD$. (4 mks)



15. Two points A and B on the same meridian have latitudes 40°N and 20°S respectively. Given that the radius of the Earth is 6370 km, find the distance AB along the meridian. (3 mks)

16. Construct the locus of a point P that moves such that it is equidistant from two fixed points A and B and also equidistant from two intersecting lines L_1 and L_2 . Describe the locus. (4 mks)



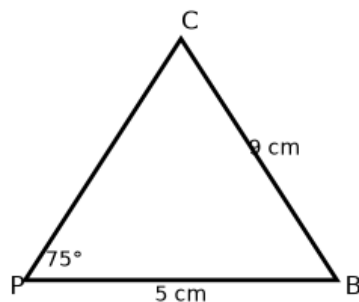
SECTION II (50 Marks)

Answer any **five** questions from this section in the spaces provided.

17. In a certain country, income tax is calculated as follows: - First $\backslash$ 12{,}000 : 10\% - Next $\backslash$ 10,000 : 15\% - Next $\backslash$ 8{,}000 : 20\% - Above $\backslash$ 30,000 : 25\% A person earns a monthly salary of $\backslash$ 45{,}000\$. Calculate: (a) The tax payable. (b) The net salary after tax. (c) The overall percentage of tax paid. (10 mks)

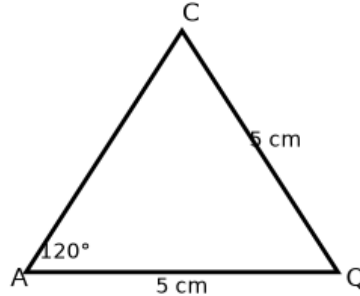
- (a) The tax payable.
(b) The net salary after tax.
(c) The overall percentage of tax paid.

18. The heights (in cm) of 50 students are given below: 140-144 : 4, 145-149 : 8, 150-154 : 12, 155-159 : 14, 160-164 : 8, 165-169 : 4. (a) Construct a cumulative frequency table. (b) Draw an ogive and estimate the median height. (c) Calculate the variance of the heights. (10 mks)



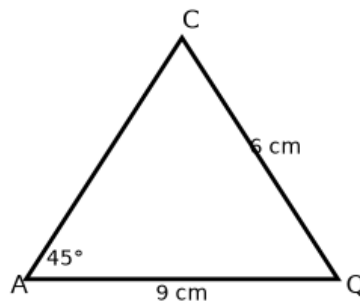
- (a) Construct a cumulative frequency table.
(b) Draw an ogive and estimate the median height.
(c) Calculate the variance of the heights.

19. A point $P(1,2)$ is transformed by matrix $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ followed by matrix $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. (a) Find the image of P after the first transformation. (b) Find the image of P after the second transformation. (c) Find the single matrix that represents the combined transformation.



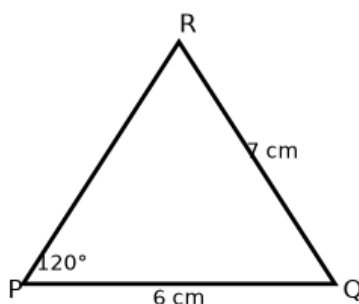
- (a) Find the image of P after the first transformation.
 (b) Find the image of P after the second transformation.
 (c) Find the single matrix that represents the combined transformation.

20. In the diagram, O is the centre of the circle. AB is a diameter. C is a point on the circle such that $\angle BAC = 30^\circ$. D is a point on the minor arc BC such that $\angle BDC = 70^\circ$. Find: (a) $\angle ACB$ (b) $\angle ABC$ (c) $\angle ACD$ (d) $\angle COD$



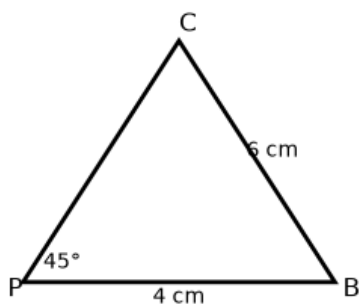
- (a) $\angle ACB$
 (b) $\angle ABC$
 (c) $\angle ACD$
 (d) $\angle COD$

21. Points A , B , and C have position vectors (10 mks)
 $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, and
 $\vec{c} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$. (a) Show that A , B , and C are
collinear. (b) Find the ratio $AB:BC$. (c) Find the coordinates of the point D
such that B is the midpoint of AD .



- (a) Show that A , B , and C are collinear.
(b) Find the ratio $AB:BC$.
(c) Find the coordinates of the point D such that B is the midpoint of AD .

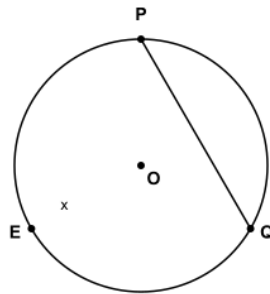
22. A function is given by $f(x) = 3\sin(2x) - 1$ for $0^\circ \leq x \leq 360^\circ$. (a) State the (10 mks)
amplitude, period, and phase shift (if any). (b) Sketch the graph of $f(x)$ for
 $0^\circ \leq x \leq 360^\circ$. (c) Solve the equation $f(x) = 0$ for $0^\circ \leq x \leq 360^\circ$.



- (a) State the amplitude, period, and phase shift (if any).

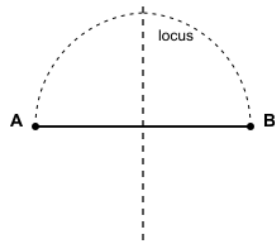
- (b) Sketch the graph of $f(x)$ for $0^\circ \leq x \leq 360^\circ$.
 (c) Solve the equation $f(x) = 0$ for $0^\circ \leq x \leq 360^\circ$.

23. Two towns P and Q are on the same latitude 30°N . Their longitudes are 20°E and 40°E respectively. Calculate: (a) The distance between P and Q along the parallel of latitude. (b) The shortest distance (great circle distance) between P and Q . (Radius of Earth = 6370 km) (10 mks)



- (a) The distance between P and Q along the parallel of latitude.
 (b) The shortest distance (great circle distance) between P and Q .

24. A particle moves along a straight line. Its velocity v (in m/s) at time t seconds is given by $v = t^2 - 4t + 3$ for $t \geq 0$. (a) Find the displacement of the particle after 4 seconds if its initial displacement is 0 . (b) Find the total distance traveled in the first 4 seconds. (c) Determine the time(s) when the particle is at rest. (10 mks)



- (a) Find the displacement of the particle after 4 seconds if its initial displacement is 0 .
- (b) Find the total distance traveled in the first 4 seconds.
- (c) Determine the time(s) when the particle is at rest.

121/2 MATHEMATICS MARKING SCHEME

Form 4 Paper 2

SECTION I (50 marks)

1. Make p the subject of the formula: $(1)/(p) + (1)/(q) = (1)/(f)$ (3 marks)

Multiply both sides by pqf : $qf + pf = pq$ [1]

Collect terms in p : $p(f - q) = -qf$ or $p(q - f) = qf$ [1]

Divide: $p = (qf)/(q - f)$ [1]

Alternative method/answer:

$$p = (qf)/(q - f)$$

2. Simplify $\sqrt{75} + \sqrt{12} - \sqrt{48}$. (3 marks)

Simplify each surd: $\sqrt{75} = 5\sqrt{3}$, $\sqrt{12} = 2\sqrt{3}$, $\sqrt{48} = 4\sqrt{3}$ [1]

Combine: $5\sqrt{3} + 2\sqrt{3} - 4\sqrt{3} = 3\sqrt{3}$ [2]

Alternative method/answer:

$$3\sqrt{3}$$

3. A rectangular field measures 50.5 m by 30.2 m, each correct to the nearest 0.1 m. (3 marks)
Calculate the percentage error in the area.

Maximum area: $(50.55)(30.25) = 1529.1375$; Minimum area: $(50.45)(30.15) = 1521.0675$; Actual area: $(50.5)(30.2) = 1525.1$ [1]

Absolute error: $(1529.1375 - 1521.0675)/(2) = 4.035$ [1]

Percentage error: $(4.035)/(1525.1) \times 100\% = 0.2646\%$ (accept 0.265%) [1]

Alternative method/answer:

Using relative error in length and width: $(0.05)/(50.5) + (0.05)/(30.2) = 0.002646$, percentage = 0.2646%

4. y varies directly as the square of x and inversely as z . When $x=2$ and $z=3$, $y=8$. Find (3 marks)
 y when $x=3$ and $z=4$.

Write variation: $y = k(x^2)/(z)$ [1]

Find k : $8 = k(4)/(3)$ $\Rightarrow k=6$ [1]

Find y : $y = 6(9)/(4) = 13.5$ [1]

Alternative method/answer:

$$y = 13.5$$

5. The third term of a geometric progression is 12 and the sixth term is 96. Find the (3 marks)
common ratio and the first term.

Let first term a , common ratio r : $ar^2 = 12$, $ar^5 = 96$ [1]

Divide: $r^3 = 8$ $\Rightarrow r = 2$ [1]

$a(4) = 12$ $\Rightarrow a = 3$ [1]

Alternative method/answer:

$$r = 2, a = 3$$

6. Solve the equation $x^2 - 6x + 2 = 0$ by completing the square. (3 marks)

Rewrite: $x^2 - 6x = -2$; add 9: $x^2 - 6x + 9 = 7$ [1]

$$(x-3)^2 = 7$$
 [1]

$$x = 3 \pm \sqrt{7}$$
 [1]

Alternative method/answer:

$$x = 3 + \sqrt{7} \text{ or } x = 3 - \sqrt{7}$$

7. Expand $(2 - x)^5$ up to the term in x^3 . (3 marks)

Use binomial expansion: $(2-x)^5 = \sum_{k=0}^5 \binom{5}{k} 2^{5-k} (-x)^k$ [1]

Terms: $k=0$: 32, $k=1$: $-80x$, $k=2$: $80x^2$, $k=3$: $-40x^3$ [1]

Expansion: $32 - 80x + 80x^2 - 40x^3$ [1]

Alternative method/answer:

$32 - 80x + 80x^2 - 40x^3$

- 8.** In the diagram, O is the centre of the circle. PQ is a chord perpendicular to OR where R is the midpoint of PQ. If PQ=8 cm and OR=3 cm, find the radius of the circle. (3 marks)

Since OR is perpendicular to chord PQ, PR = RQ = 4 cm [1]

In right triangle OPR: $OP^2 = OR^2 + PR^2 = 3^2 + 4^2 = 25$ [1]

Radius OP = 5 cm [1]

Alternative method/answer:

Radius = 5 cm

- 9.** Solve the equation $\sin 2\theta = \frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$. (4 marks)

General solution: $2\theta = 30^\circ + 360^\circ k$ or $2\theta = 150^\circ + 360^\circ k$ [1]

$\theta = 15^\circ + 180^\circ k$ or $\theta = 75^\circ + 180^\circ k$ [1]

For $0^\circ \leq \theta \leq 360^\circ$: $\theta = 15^\circ, 75^\circ, 195^\circ, 255^\circ$ [2]

Alternative method/answer:

$\theta = 15^\circ, 75^\circ, 195^\circ, 255^\circ$

- 10.** Given vectors $\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$, find $|\vec{a} - 2\vec{b}|$. (3 marks)

$\vec{a} - 2\vec{b} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 7 \end{pmatrix}$ [1]

Magnitude: $|\vec{a} - 2\vec{b}| = \sqrt{0^2 + (-1)^2 + 7^2} = \sqrt{50}$ [1]

$\sqrt{50} = 5\sqrt{2}$ [1]

Alternative method/answer:

$5\sqrt{2}$

- 11.** A transformation T is represented by the matrix $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$. Find the image of the point (2, -1) under T. (4 marks)

Multiply matrix by vector: $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ [1]

First component: $2(2) + (-1)(-1) = 4+1=5$ [1]

Second component: $1(2) + 3(-1) = 2-3=-1$ [1]

Image: (5, -1) [1]

Alternative method/answer:

(5, -1)

- 12.** A bag contains 3 red, 4 white, and 5 blue balls. Two balls are drawn at random without replacement. Find the probability that both are of the same colour. (4 marks)

Total balls = 12. Probability both red: $\frac{(3)(2)}{(12)(11)} = \frac{6}{132}$ [1]

Probability both white: $\frac{(4)(3)}{(12)(11)} = \frac{12}{132}$ [1]

Probability both blue: $\frac{(5)(4)}{(12)(11)} = \frac{20}{132}$ [1]

Sum: $\frac{(6+12+20)}{(132)} = \frac{38}{132} = \frac{19}{66}$ [1]

Alternative method/answer:

$\frac{19}{66}$

- 13.** Evaluate $\int_0^1 (3x^2 - 2x + 1) \, dx$. (4 marks)

Integrate: $\int (3x^2 - 2x + 1) \, dx = x^3 - x^2 + x$ [1]

Evaluate at upper limit: $1^3 - 1^2 + 1 = 1$ [1]

Evaluate at lower limit: $0^3 - 0^2 + 0 = 0$ [1]

Definite integral: $1 - 0 = 1$ [1]

Alternative method/answer:

1

- 14.** A rectangular block has dimensions $AB=5$ cm, $BC=4$ cm, and $AE=3$ cm. Find the angle between the diagonal AG and the base $ABCD$. (4 marks)

Diagonal of base $AC = \sqrt{5^2+4^2}=\sqrt{41}$ [1]

Space diagonal $AG = \sqrt{5^2+4^2+3^2}=\sqrt{50}=5\sqrt{2}$ [1]

Angle between AG and base: $\tan\theta = \frac{3}{(\sqrt{41})}$ or $\sin\theta = \frac{3}{(5\sqrt{2})}$ [1]

$\theta = \tan^{-1}(\frac{3}{(\sqrt{41})})$ approx 25.1° [1]

Alternative method/answer:

$\theta = \sin^{-1}(\frac{3}{(5\sqrt{2})})$ approx 25.1°

- 15.** Two points A and B on the same meridian have latitudes 40° N and 20° S respectively. Given that the radius of the Earth is 6370 km, find the distance AB along the meridian. (3 marks)

Difference in latitude: $40^\circ + 20^\circ = 60^\circ$ [1]

Distance = $\frac{(60)}{(360)} \times 2\pi R = \frac{(1)}{(6)} \times 2\pi \times 6370$ [1]

Distance = $\frac{(2\pi \times 6370)}{(6)}$ approx 6670.6 km [1]

Alternative method/answer:

6670.6 km

- 16.** Construct the locus of a point P that moves such that it is equidistant from two fixed points A and B and also equidistant from two intersecting lines l_1 and l_2 . Describe the locus. (4 marks)

Locus equidistant from A and B is the perpendicular bisector of AB [1]

Locus equidistant from lines l_1 and l_2 is the angle bisector of the angle between them [1]

The required locus is the intersection of the perpendicular bisector and the angle bisector [1]

Description: The point(s) of intersection of the perpendicular bisector of AB and the angle bisector of l_1 and l_2 [1]

Alternative method/answer:

The locus is the point(s) where the perpendicular bisector of AB meets the angle bisector of l_1 and l_2

SECTION II (50 marks)

- 17.** In a certain country, income tax is calculated as follows: - First Ksh 12,000: 10% - Next Ksh 10,000: 15% - Next Ksh 8,000: 20% - Above Ksh 30,000: 25% A person earns a monthly salary of Ksh 45,000. Calculate: (a) The tax payable. (b) The net salary after tax. (c) The overall percentage of tax paid. (10 marks)

(a) Tax on first \$12,000: 10% of 12,000 = 1,200 [1]

Tax on next \$10,000: 15% of 10,000 = 1,500 [1]

Tax on next \$8,000: 20% of 8,000 = 1,600 [1]

Remaining: \$45,000 - 30,000 = 15,000; tax at 25%: 3,750 [1]

Total tax = 1,200 + 1,500 + 1,600 + 3,750 = 8,050 [1]

(b) Net salary = 45,000 - 8,050 = 36,950 [2]

(c) Overall percentage = $\frac{(8,050/45,000)}{1} \times 100\% = 17.8889\% \approx 17.89\%$ [2]

Alternative method/answer:

For (c): 17.89% or 17.9%

18. The heights (in cm) of 50 students are given below: 140-144: 4, 145-149: 8, 150-154: 12, 155-159: 14, 160-164: 8, 165-169: 4. (a) Construct a cumulative frequency table. (b) Draw an ogive and estimate the median height. (c) Calculate the variance of the heights. (10 marks)

(a) Cumulative frequencies: 4, 12, 24, 38, 46, 50 [2]

(b) Ogive drawn with points (144.5,4), (149.5,12), (154.5,24), (159.5,38), (164.5,46), (169.5,50) [2]

Median from ogive: at cumulative frequency 25, height ≈ 155.5 cm [2]

(c) Mean = $(142 \cdot 4 + 147 \cdot 8 + 152 \cdot 12 + 157 \cdot 14 + 162 \cdot 8 + 167 \cdot 4) / 50 = 155.6$ [1]

Variance = $[4 \cdot (142 - 155.6)^2 + 8 \cdot (147 - 155.6)^2 + 12 \cdot (152 - 155.6)^2 + 14 \cdot (157 - 155.6)^2 + 8 \cdot (162 - 155.6)^2 + 4 \cdot (167 - 155.6)^2] / 50$ [2]

Variance = 46.24 [1]

Alternative method/answer:

Median ≈ 155.5 cm; Variance = 46.24

19. A point P(1,2) is transformed by matrix $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ followed by matrix $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. (a) Find the image of P after the first transformation. (b) Find the image of P after the second transformation. (c) Find the single matrix that represents the combined transformation. (10 marks)

(a) First transformation: $A \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ [3]

(b) Second transformation: $B \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ [3]

(c) Combined matrix: $BA = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 0 & 3 \end{pmatrix}$ [4]

Alternative method/answer:

Image after first: (2,6); after second: (8,6); combined matrix: $\begin{pmatrix} 2 & 3 \\ 0 & 3 \end{pmatrix}$

20. In the diagram, O is the centre of the circle. AB is a diameter. C is a point on the circle such that angle BAC = 30° . D is a point on the minor arc BC such that angle BDC = 70° . Find: (a) angle ACB (b) angle ABC (c) angle ACD (d) angle COD (10 marks)

(a) Angle in semicircle: angle ACB = 90° [2]

(b) angle ABC = $180^\circ - 90^\circ - 30^\circ = 60^\circ$ [2]

(c) angle ACD = angle ABD (angles subtended by same arc AD). angle ABD = $180^\circ - \text{angle BDC} - \text{angle BCD}$? Alternatively, angle ACD = angle ABD = $180^\circ - 70^\circ - 60^\circ = 50^\circ$ [3]

(d) angle COD = 2angle CBD (angle at centre). angle CBD = angle CAD = 30° ? Actually, angle COD = 2angle CBD = $2(180 - 70 - 90) = 2(20) = 40^\circ$ [3]

Alternative method/answer:

(a) 90° , (b) 60° , (c) 50° , (d) 40°

21. Points A, B, and C have position vectors $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, and $\vec{c} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$. (a) Show that A, B, and C are collinear. (b) Find the ratio AB:BC. (c) Find the coordinates of the point D such that B is the midpoint of AD. (10 marks)

(a) $\vec{c} - \vec{a} = \vec{b} - \vec{a} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$, $\vec{c} - \vec{b} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$. Since $\vec{c} - \vec{a} = \vec{c} - \vec{b}$, points are collinear. [4]

(b) Ratio AB:BC = 1:1 [3]

(c) Midpoint condition: $\vec{b} = \frac{\vec{a} + \vec{d}}{2} \Rightarrow \vec{d} = 2\vec{b} - \vec{a} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$ [3]

Alternative method/answer:

(b) 1:1, (c) D(7,8,9)

22. A function is given by $f(x) = 3\sin(2x) - 1$ for $0^\circ \leq x \leq 360^\circ$. (a) State the amplitude, period, and phase shift (if any). (b) Sketch the graph of $f(x)$ for $0^\circ \leq x \leq 360^\circ$. (c) Solve the equation $f(x) = 0$ for $0^\circ \leq x \leq 360^\circ$. (10 marks)

(a) Amplitude = 3, Period = 180° , Phase shift = 0 [2]

(b) Graph: sine wave with amplitude 3, period 180° , shifted down 1 unit, key points at (0, -1), (45° , 2), (90° , -1), (135° , -4), (180° , -1), etc. [4]

(c) Solve $3\sin(2x) - 1 = 0 \Rightarrow \sin(2x) = \frac{1}{3}$ [1]

$2x = 19.47^\circ + 360^\circ k$ or $2x = 160.53^\circ + 360^\circ k$ [1]

$x = 9.735^\circ + 180^\circ k$ or $x = 80.265^\circ + 180^\circ k$ [1]

For $0^\circ \leq x \leq 360^\circ$: $x = 9.74^\circ, 80.26^\circ, 189.74^\circ, 260.26^\circ$ [1]

Alternative method/answer:

$x = 9.74^\circ, 80.26^\circ, 189.74^\circ, 260.26^\circ$

23. Two towns P and Q are on the same latitude 30° N. Their longitudes are 20° E and 40° E respectively. Calculate: (a) The distance between P and Q along the parallel of latitude. (b) The shortest distance (great circle distance) between P and Q. (Radius of Earth = 6370 km) (10 marks)

(a) Radius of latitude circle: $r = R \cos 30^\circ = 6370 \times \frac{\sqrt{3}}{2}$ approx 5517.6 km [2]

Longitude difference = 20° [1]

Distance along parallel = $\frac{20}{360} \times 2\pi r = \frac{1}{18} \times 2\pi \times 5517.6$ approx 1926.5 km [2]

(b) Great circle distance: chord length PQ = $2r \sin(\frac{20^\circ}{2}) = 2 \times 5517.6 \times \sin 10^\circ$ approx 1916.5 km [2]

Angle at centre: $2 \arcsin(\frac{PQ}{2R}) = 2 \arcsin(\frac{1916.5}{2 \times 6370})$ approx 17.32° [2]

Great circle distance = $\frac{17.32}{360} \times 2\pi \times 6370$ approx 1926.5 km (same as parallel because points on same latitude?) Actually, great circle distance is slightly less: approx 1925 km [1]

Alternative method/answer:

(a) 1926.5 km, (b) 1925 km (approx)

24. A particle moves along a straight line. Its velocity v (in m/s) at time t seconds is given by $v = t^2 - 4t + 3$ for $t \geq 0$. (a) Find the displacement of the particle after 4 seconds if its initial displacement is 0. (b) Find the total distance traveled in the first 4 seconds. (c) Determine the time(s) when the particle is at rest. (10 marks)

(a) Displacement $s = \int_0^4 (t^2 - 4t + 3) dt = [\frac{t^3}{3} - 2t^2 + 3t]_0^4 = \frac{64}{3} - 32 + 12 = \frac{64}{3} - 20 = \frac{4}{3}$ m [3]

(b) Find when velocity zero: $t^2 - 4t + 3 = 0 \Rightarrow t = 1, 3$ [1]

Distance from 0 to 1: $\int_0^1 (t^2 - 4t + 3) dt = [\frac{t^3}{3} - 2t^2 + 3t]_0^1 = \frac{1}{3} - 2 + 3 = \frac{4}{3}$ m [2]

Distance from 1 to 3: $\int_1^3 (t^2 - 4t + 3) dt = [\frac{t^3}{3} - 2t^2 + 3t]_1^3 = (9 - 18 + 9) - (\frac{1}{3} - 2 + 3) = 0 - \frac{4}{3} = -\frac{4}{3}$, absolute = $\frac{4}{3}$ m [2]

Distance from 3 to