

EDUZEDCO
121/2 MATHEMATICS
Paper 2
Form 1 End Term Examination

Time: 2 ½ Hours

Name: Adm No:
 School: Class:
 Signature: Date:

INSTRUCTIONS TO CANDIDATES

- (a) Write your name and index number in the spaces provided at the top of this page.
- (b) Write your school name, sign and write the date of the examination in the spaces provided above.
- (c) This paper consists of **Two** sections: **Section I** and **Section II**.
- (d) Answer **ALL** questions in Section I and any **five questions** from Section II.
- (e) Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question.
- (f) Marks may be given for correct working even if the answer is wrong.
- (g) Non-Programmable silent electronic calculators and KNEC Mathematical Tables may be used.
- (h) This paper consists of **8 printed pages**.
- (i) Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.
- (j) Candidates should answer the questions in English.

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

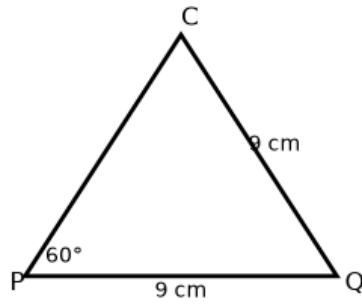
SECTION II

17	18	19	20	21	22	23	24	Total	Grand Total

SECTION I (50 Marks)

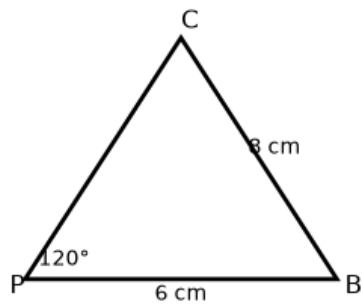
Answer **all** questions in this section in the spaces provided.

1. Make r the subject of the formula $V = \frac{4}{3}\pi r^2 h$. (2 mks)
2. Simplify $\frac{3}{(\sqrt{5}-2)}$ in the form $a + b\sqrt{c}$ where a , b , and c are integers. (3 mks)
3. A rectangular field measures 45.6 m by 28.3 m, correct to one decimal place. Calculate the percentage error in the area. (3 mks)
4. y varies jointly as x^2 and z . When $x=3$ and $z=4$, $y=36$. Find y when $x=4$ and $z=5$. (3 mks)
5. The third term of a geometric progression is 36 and the sixth term is 972 . Find the first term and the common ratio. (3 mks)
6. Solve the equation $2x^2 - 5x - 3 = 0$ by completing the square. (3 mks)
7. Expand $(1-2x)^5$ in ascending powers of x up to the term in x^3 . (3 mks)
8. In the diagram below, O is the centre of the circle. AB is a chord of length 8 cm and OM is perpendicular to AB with $OM=3$ cm. Find the radius of the circle. (3 mks)

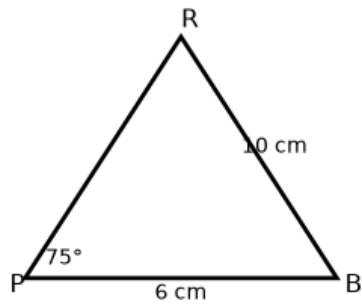


9. Solve $\cos 2x = 0.5$ for $0^\circ \leq x \leq 360^\circ$. (3 mks)

10. Given vectors $\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$, find $|\vec{a} - \vec{b}|$. (4 mks)



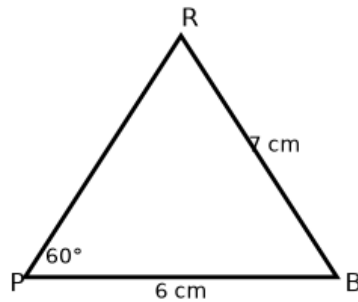
11. A transformation is represented by matrix $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$. Find the image of the point $(3, -2)$ under this transformation. (4 mks)



12. A bag contains 5 red balls and 3 blue balls. Two balls are drawn without replacement. Find the probability that both balls are red. (4 mks)

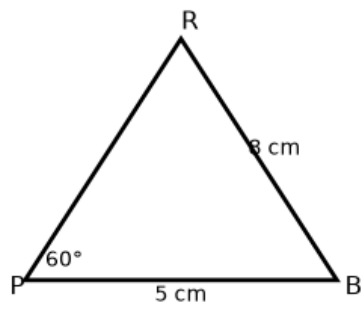
13. Evaluate $\int_0^2 (3x^2 - 2x + 1) \, dx$. (4 mks)

14. A rectangular box has dimensions 4 cm by 3 cm by 2 cm. Find the angle between the diagonal of the base and the diagonal of the box. (4 mks)



15. Two points A and B on the same longitude have latitudes 40°N and 10°S respectively. If the Earth's radius is 6370 km, find the distance between A and B along the longitude. (Take $\pi = 3.142$) (4 mks)

16. Construct the locus of points equidistant from two fixed points P and Q and also equidistant from two intersecting lines l_1 and l_2 . Describe the resulting point. (4 mks)



SECTION II (50 Marks)

Answer any **five** questions from this section in the spaces provided.

- 17.** In a certain country, income tax is calculated as follows: - First \ 120,000 : no tax \ 100,000 : 10% - Next \ 200,000 : 20% - Next \ 300,000 : 30% - Next \ 720,000 : 40% A person earns a monthly salary of \ 50,000 and is entitled to a monthly personal relief of \ 2,000\$. Calculate: (a) The annual taxable income. (b) The total annual tax paid. (c) The net monthly income after tax. (10 mks)

The annual taxable income.

The total annual tax paid.

The net monthly income after tax.

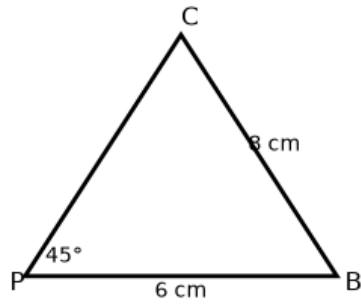
- 18.** The table below shows the distribution of marks obtained by 50 students in a test. | Marks | Frequency | |-----|-----| | 10-19 | 4 | | 20-29 | 8 | | 30-39 | 12 | | 40-49 | 16 | | 50-59 | 6 | | 60-69 | 4 | (a) Construct a cumulative frequency table and draw an ogive. (b) Using the ogive, estimate the median mark. (c) Calculate the variance of the marks. (10 mks)

Construct a cumulative frequency table and draw an ogive.

Using the ogive, estimate the median mark.

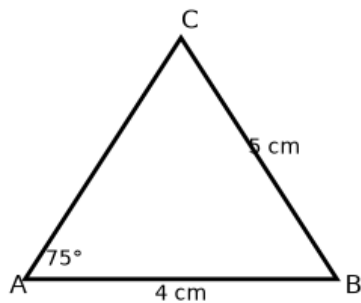
Calculate the variance of the marks.

- 19.** A triangle with vertices $A(1,0)$, $B(2,1)$, and $C(1,2)$ is transformed by a matrix $M = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ followed by a matrix $N = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. (a) Find the image of the triangle after the combined transformation. (b) Describe the transformation represented by M and N individually. (c) Find the single matrix that represents the combined transformation. (10 mks)



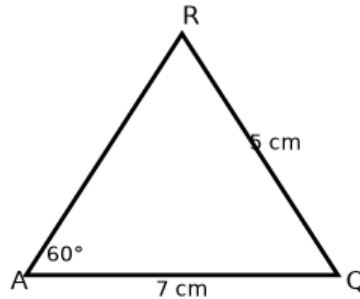
Find the image of the triangle after the combined transformation.
 Describe the transformation represented by M and N individually.
 Find the single matrix that represents the combined transformation.

20. In the diagram below, O is the centre of the circle. $\angle AOB = 80^\circ$ and $\angle BOC = 50^\circ$. Points A, B, C, and D lie on the circle. AD and BC intersect at E. (a) Find $\angle ABC$. (b) Find $\angle ADC$. (c) Find $\angle AEC$. (10 mks)



Find $\angle ABC$.
 Find $\angle ADC$.
 Find $\angle AEC$.

21. Points P , Q , and R have position vectors $\vec{p} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{q} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, and $\vec{r} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$. (a) Show that P , Q , and R are collinear. (b) Find the ratio $PQ:QR$. (c) Find the coordinates of the point S such that $\vec{os} = \vec{op} + \frac{2}{3}\vec{pq}$. (10 mks)

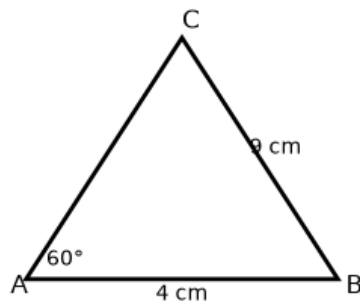


Show that P , Q , and R are collinear.

Find the ratio $PQ:QR$.

Find the coordinates of S such that $\vec{os} = \vec{op} + \frac{2}{3}\vec{pq}$.

22. The function $f(x) = 3 \sin(2x - 30^\circ)$ is defined for $0^\circ \leq x \leq 180^\circ$. (a) Determine the amplitude and period of f . (b) Sketch the graph of f for the given domain. (c) Solve $f(x) = 1.5$ for $0^\circ \leq x \leq 180^\circ$. (10 mks)

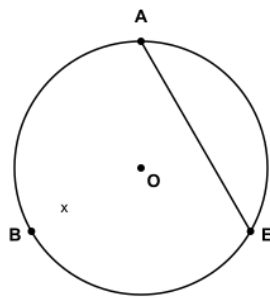


Determine the amplitude and period of f .

Sketch the graph of f for the given domain.

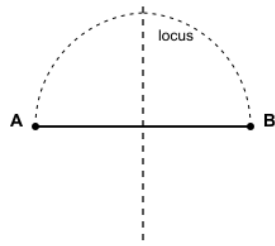
Solve $f(x) = 1.5$ for $0^\circ \leq x \leq 180^\circ$.

23. Two towns A (60°N , 30°E) and B (60°N , 90°E) are on the same latitude. The radius of the Earth is 6370 km. (Take $\pi = 3.142$) (a) Find the distance between A and B along the parallel of latitude. (b) An aircraft flies from A to B along the great circle route. Find the shortest distance between A and B . (c) If the aircraft flies at a speed of 800 km/h, how long does the flight take? (10 mks)



Find the distance between A and B along the parallel of latitude.
Find the shortest distance between A and B along the great circle.
If the aircraft flies at 800 km/h, how long does the flight take?

24. A particle moves along a straight line such that its velocity v m/s at time t seconds is given by $v = 3t^2 - 12t + 9$ for $t \geq 0$. (a) Find the initial velocity. (b) Find the time when the particle is momentarily at rest. (c) Find the displacement of the particle from its starting point after 3 seconds. (d) Find the total distance traveled in the first 3 seconds. (10 mks)



Find the initial velocity.

Find the time when the particle is momentarily at rest.

Find the displacement after 3 seconds.

Find the total distance traveled in the first 3 seconds.

121/2 MATHEMATICS MARKING SCHEME

Form 1 Paper 2

SECTION I (50 marks)

1. Make r the subject of the formula $V = \frac{4}{3}\pi r^2 h$. (2 marks)

Multiply both sides by 3: $3V = 4\pi r^2 h$ [1]

Divide both sides by $4\pi h$: $r^2 = \frac{3V}{4\pi h}$ [0.5]

Take square root: $r = \sqrt{\frac{3V}{4\pi h}}$ [0.5]

Alternative method/answer:

$$r = \sqrt{\frac{3V}{4\pi h}}$$

2. Simplify $\frac{3}{\sqrt{5} - 2}$ in the form $a + b\sqrt{c}$ where a , b , and c are integers. (3 marks)

Rationalize denominator: $\frac{3}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$ [1]

Simplify numerator: $3(\sqrt{5}+2) = 3\sqrt{5}+6$ [1]

Simplify denominator: $(\sqrt{5})^2 - 2^2 = 5-4=1$, so answer $6+3\sqrt{5}$ [1]

Alternative method/answer:

$$6+3\sqrt{5}$$

3. A rectangular field measures 45.6 m by 28.3 m, correct to one decimal place. Calculate the percentage error in the area. (3 marks)

Maximum area: $(45.65)(28.35) = 1294.1775$; minimum area: $(45.55)(28.25) = 1286.7875$ [1]

Actual area (using given): $45.6 \times 28.3 = 1290.48$; absolute error =

$$\frac{1294.1775-1286.7875}{2} = 3.695$$
 [1]

Percentage error = $\frac{3.695}{1290.48} \times 100 \approx 0.286\%$ [1]

Alternative method/answer:

Using formula: $\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta w}{w} =$

$$\frac{0.05}{45.6} + \frac{0.05}{28.3} = 0.001096 + 0.001767 = 0.002863, \text{ percentage} = 0.286\%$$

4. y varies jointly as x^2 and z . When $x = 3$ and $z = 4$, $y = 36$. Find y when $x = 4$ and $z = 5$. (3 marks)

Write variation: $y = kx^2z$ [1]

Find k : $36 = k(3^2)(4) = 36k \Rightarrow k=1$ [1]

Find y : $y = 1 \times 4^2 \times 5 = 80$ [1]

Alternative method/answer:

$$y=80$$

5. The third term of a geometric progression is 36 and the sixth term is 972. Find the first term and the common ratio. (3 marks)

Let first term = a , common ratio = r . Then $ar^2=36$, $ar^5=972$ [1]

Divide: $\frac{ar^5}{ar^2} = r^3 = \frac{972}{36} = 27 \Rightarrow r=3$ [1]

Substitute: $a(3^2)=36 \Rightarrow 9a=36 \Rightarrow a=4$ [1]

Alternative method/answer:

$$a=4, r=3$$

6. Solve the equation $2x^2 - 5x - 3 = 0$ by completing the square. (3 marks)

Divide by 2: $x^2 - \frac{5}{2}x - \frac{3}{2} = 0$ [0.5]

Add $\left(\frac{5}{4}\right)^2 = \frac{25}{16}$ to both sides: $x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{3}{2} + \frac{25}{16} = \frac{24}{16} + \frac{25}{16} = \frac{49}{16}$ [1]

Write as perfect square: $\left(x - \frac{5}{4}\right)^2 = \frac{49}{16}$ [0.5]

Take square root: $x - \frac{5}{4} = \pm \frac{7}{4} \Rightarrow x = \frac{5}{4} \pm \frac{7}{4} \Rightarrow x = 3 \text{ or } x = -\frac{1}{2}$ [1]

Alternative method/answer:

$$x = 3 \text{ or } x = -0.5$$

7. Expand $(1 - 2x)^5$ in ascending powers of x up to the term in x^3 . (3 marks)

Use binomial theorem: $(1 - 2x)^5 = \sum_{k=0}^5 \binom{5}{k} (1)^{5-k} (-2x)^k$ [1]

Compute terms: $k=0: 1$, $k=1: 5(-2x) = -10x$, $k=2: 10(4x^2) = 40x^2$, $k=3: 10(-8x^3) = -80x^3$ [1.5]

Write expansion: $1 - 10x + 40x^2 - 80x^3$ [0.5]

Alternative method/answer:

$$1 - 10x + 40x^2 - 80x^3$$

8. In the diagram below, O is the centre of the circle. AB is a chord of length 8 cm and OM is perpendicular to AB with $OM = 3$ cm. Find the radius of the circle. (3 marks)

Since OM is perpendicular to AB , M is midpoint of AB . So $AM = 4$ cm. [1]

In right triangle OMA , $OA^2 = OM^2 + AM^2 = 3^2 + 4^2 = 9 + 16 = 25$ [1]

Radius $OA = \sqrt{25} = 5$ cm [1]

Alternative method/answer:

$$5 \text{ cm}$$

9. Solve $\cos 2x = 0.5$ for $0^\circ \leq x \leq 360^\circ$. (3 marks)

General solution: $\cos 2x = 0.5 \Rightarrow 2x = 60^\circ + 360^\circ n \text{ or } 2x = 300^\circ + 360^\circ n$ [1]

Divide by 2: $x = 30^\circ + 180^\circ n \text{ or } x = 150^\circ + 180^\circ n$ [1]

For $0^\circ \leq x \leq 360^\circ$ [1]

Alternative method/answer:

$$30^\circ, 150^\circ, 210^\circ, 330^\circ$$

10. Given vectors $\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$, find $|\vec{a} - \vec{b}|$. (4 marks)

Compute $\vec{a} - \vec{b} = \begin{pmatrix} 2-1 \\ -1-4 \\ 3-(-2) \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ 5 \end{pmatrix}$ [2]

Magnitude: $|\vec{a} - \vec{b}| = \sqrt{1^2 + (-5)^2 + 5^2} = \sqrt{1+25+25} = \sqrt{51}$ [2]

Alternative method/answer:

$$\sqrt{51}$$

11. A transformation is represented by matrix $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$. Find the image of the point $(3, -2)$ under this transformation. (4 marks)

Write transformation: $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ [1]

Multiply: $x' = 2(3) + (-1)(-2) = 6 + 2 = 8$ [1.5]

$y' = 1(3) + 3(-2) = 3 - 6 = -3$ [1.5]

Alternative method/answer:

$$\text{Image point: } (8, -3)$$

12. A bag contains 5 red balls and 3 blue balls. Two balls are drawn without replacement. Find the probability that both balls are red. (4 marks)

Probability first red: $\frac{5}{8}$ [1]

Probability second red given first red: $\frac{4}{7}$ [1]

Multiply: $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$ [1]

Simplify: $\frac{5}{14}$ [1]

Alternative method/answer:

$$\frac{5}{14}$$

13. Evaluate $\int_0^2 (3x^2 - 2x + 1) \, dx$. (4 marks)

Integrate: $\int (3x^2 - 2x + 1) \, dx = x^3 - x^2 + x$ [2]

Evaluate from 0 to 2: $[2^3 - 2^2 + 2] - [0] = 8 - 4 + 2 = 6$ [2]

Alternative method/answer:

6

14. A rectangular box has dimensions 4 cm by 3 cm by 2 cm. Find the angle between the diagonal of the base and the diagonal of the box. (4 marks)

Diagonal of base: $d_b = \sqrt{4^2 + 3^2} = 5$ cm [1]

Diagonal of box: $d = \sqrt{4^2 + 3^2 + 2^2} = \sqrt{29}$ cm [1]

Angle between them: $\cos \theta = \frac{d_b}{d} = \frac{5}{\sqrt{29}}$ [1]

$\theta = \cos^{-1} \left(\frac{5}{\sqrt{29}} \right) \approx 21.8^\circ$ [1]

Alternative method/answer:

$\theta = \cos^{-1} \left(\frac{5}{\sqrt{29}} \right)$ or 21.8°

15. Two points A and B on the same longitude have latitudes 40°N and 10°S respectively. If the Earth's radius is 6370 km, find the distance between A and B along the longitude. (Take $\pi = 3.142$) (4 marks)

Angular difference: 40°N to $10^\circ\text{S} = 50^\circ$ [1]

Distance = $\frac{\theta}{360} \times 2\pi R = \frac{50}{360} \times 2 \times 3.142 \times 6370$ [1.5]

Calculate: $\frac{50}{360} \times 2 \times 3.142 \times 6370 = \frac{50}{360} \times 40038.68 \approx 5560.93$ km [1.5]

Alternative method/answer:

5561 km (approx)

16. Construct the locus of points equidistant from two fixed points P and Q and also equidistant from two intersecting lines l_1 and l_2 . Describe the resulting point. (4 marks)

Locus equidistant from P and Q: perpendicular bisector of PQ [1]

Locus equidistant from lines l_1 and l_2 : angle bisectors of the angles formed by l_1 and l_2 [1]

Intersection: the point(s) where the perpendicular bisector meets the angle bisectors [1]

Result: one point (if lines intersect) or two points (if lines are parallel) - usually one point inside the angle [1]

Alternative method/answer:

The intersection point(s) of the perpendicular bisector and the angle bisectors.

SECTION II (50 marks)

17. In a certain country, income tax is calculated as follows: - First \$120,000: no tax - Next \$100,000: 10% - Next \$200,000: 20% - Next \$300,000: 30% - Above \$720,000: 40% A person earns a monthly salary of \$50,000 and is entitled to a monthly personal relief of \$2,000. Calculate: (a) The annual taxable income. (b) The total annual tax paid. (c) The net monthly income after tax. (10 marks)

Annual salary = $50,000 \times 12 = 600,000$ [1]

Annual taxable income = $600,000 - 2,000 \times 12 = 600,000 - 24,000 = 576,000$ [1]

Tax calculation: First \$120

Total annual tax = $10,000 + 40,000 + 46,800 = 96,800$ [1]

Monthly tax = $96,800 / 12 = 8,066.67$ [1]

Net monthly income = $50,000 - 8,066.67 = 41,933.33$ [2]

Alternative method/answer:

If personal relief is subtracted after tax: Tax before relief = \$96,800, relief = $2,000 \times 12 = 24,000$, tax after relief = \$72,800, monthly tax = \$6,066.67, net = \$43,933.33

18. The table below shows the distribution of marks obtained by 50 students in a test. | (10 marks)

Marks | Frequency | |-----|-----| | 10-19 | 4 | | 20-29 | 8 | | 30-39 | 12 | | 40-49 | 16 | | 50-59 | 6 | | 60-69 | 4 |

(a) Construct a cumulative frequency table and draw an ogive.

(b) Using the ogive, estimate the median mark. (c) Calculate the variance of the marks.

Cumulative frequency table: (10-19:4), (20-29:12), (30-39:24), (40-49:40), (50-59:46), (60-69:50) [2]

Ogive: plot upper boundaries (19.5,4), (29.5,12), (39.5,24), (49.5,40), (59.5,46), (69.5,50) and join smoothly [2]

Median: $\frac{50}{2} = 25$ th value, from ogive read mark ≈ 40 [2]

Variance: find midpoints: 14.5, 24.5, 34.5, 44.5, 54.5, 64.5; mean =

$$\frac{4(14.5) + 8(24.5) + 12(34.5) + 16(44.5) + 6(54.5) + 4(64.5)}{50} = \frac{58 + 196 + 414 + 712 + 327 + 258}{50} = \frac{1965}{50} = 39.3$$
 [2]

Variance = $\frac{\sum f(x - \bar{x})^2}{n}$ =

$$\frac{4(14.5 - 39.3)^2 + 8(24.5 - 39.3)^2 + 12(34.5 - 39.3)^2 + 16(44.5 - 39.3)^2 + 6(54.5 - 39.3)^2 + 4(64.5 - 39.3)^2}{50} = \frac{4(615.04) + 8(219.04) + 12(23.04) + 16(27.04) + 6(231.04) + 4(635.04)}{50} = \frac{2460.16 + 1752.32 + 276.48 + 432.64 + 1386.24 + 2540.16}{50} = \frac{8848}{50} = 176.96$$
 [2]

Alternative method/answer:

$$\text{Variance} = 176.96$$

19. A triangle with vertices $A(1,0)$, $B(2,1)$, and $C(1,2)$ is transformed by a (10 marks)

matrix $M = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ followed by a matrix $N = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. (a) Find the image of the triangle after the combined transformation. (b) Describe the transformation represented by M and N individually. (c) Find the single matrix that represents the combined transformation.

Combined matrix: $NM = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 2 & 0 \end{pmatrix}$ [2]

Apply to vertices: $A(1,0) \rightarrow (0,2)$, $B(2,1) \rightarrow (-1,4)$, $C(1,2) \rightarrow (-2,2)$ [3]

M is a stretch parallel to x-axis by factor 2; N is a rotation by 90° anticlockwise [2]

Single matrix: $\begin{pmatrix} 0 & -1 \\ 2 & 0 \end{pmatrix}$ [3]

Alternative method/answer:

Image: $A'(0,2)$, $B'(-1,4)$, $C'(-2,2)$

20. In the diagram below, O is the centre of the circle. $\angle AOB = 80^\circ$ and (10 marks)

$\angle BOC = 50^\circ$. Points A , B , C , and D lie on the circle. AD and BC intersect at E . (a) Find $\angle ABC$. (b) Find $\angle ADC$. (c) Find $\angle AEC$.

$\angle ABC = \frac{1}{2} \angle AOC$ (angle at circumference half angle at centre). $\angle AOC = 80 + 50 = 130^\circ$, so $\angle ABC = 65^\circ$ [3]

$\angle ADC = \frac{1}{2} \angle AOC$ (subtended by same arc AC) = 65° [3]

In triangle AEC, $\angle AEC = 180^\circ - \angle EAC - \angle ECA$. $\angle EAC = \angle DAC = \frac{1}{2} \angle DOC$? Alternatively, use cyclic quadrilateral: $\angle AEC = 180^\circ - \angle ABC = 115^\circ$ (since opposite angles of cyclic quadrilateral sum to 180). [4]

Alternative method/answer:

$$\angle AEC = 115^\circ$$

21. Points P , Q , and R have position vectors $\vec{p} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{q} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, and (10 marks)

$\vec{r} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$. (a) Show that P , Q , and R are collinear. (b) Find the ratio $PQ:QR$. (c) Find the coordinates of the point S such that $\vec{OS} = \vec{OP} + \frac{2}{3} \vec{PQ}$.

Find vectors: $\vec{PQ} = \vec{q} - \vec{p} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$, $\vec{PR} = \vec{r} - \vec{p} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} = 2\vec{PQ}$, so collinear [3]

Ratio $PQ:QR = 1:1$ (since $\vec{PQ} = \vec{QR}$) because $\vec{QR} = \vec{r} - \vec{q} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$ [3]
 $\vec{OS} = \vec{OP} + \frac{2}{3}\vec{PQ} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 1+2 \\ 2+2 \\ 3+2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$, so $S = (3,4,5)$ [4]

Alternative method/answer:

$$S = (3,4,5)$$

- 22.** The function $f(x) = 3 \sin(2x - 30^\circ)$ is defined for $0^\circ \leq x \leq 180^\circ$. (a) Determine the amplitude and period of f . (b) Sketch the graph of f for the given domain. (c) Solve $f(x) = 1.5$ for $0^\circ \leq x \leq 180^\circ$. (10 marks)

Amplitude = 3, Period = $\frac{360}{2} = 180^\circ$ [2]

Sketch: correct shape, amplitude 3, period 180, phase shift 15° to the right (since $2x-30=0 \Rightarrow x=15^\circ$), key points at $15^\circ, 105^\circ$, etc. [4]

Solve $3 \sin(2x-30) = 1.5 \Rightarrow \sin(2x-30) = 0.5 \Rightarrow 2x-30 = 30+360n$ or $150+360n$ [2]

Thus $x = 30+180n$ or $x = 90+180n$; in domain: $x = 30^\circ, 90^\circ$ [2]

Alternative method/answer:

$$x = 30^\circ, 90^\circ$$

- 23.** Two towns A ($60^\circ N, 30^\circ E$) and B ($60^\circ N, 90^\circ E$) are on the same latitude. The radius of the Earth is 6370 km. (Take $\pi = 3.142$) (10 marks)
 (a) Find the distance between A and B along the parallel of latitude. (b) An aircraft flies from A to B along the great circle route. Find the shortest distance between A and B. (c) If the aircraft flies at a speed of 800 km/h, how long does the flight take?

Radius of latitude circle: $R \cos \theta = 6370 \cos 60^\circ = 6370 \times 0.5 = 3185$ km [2]

Longitude difference: $90-30=60^\circ$; distance along parallel = $\frac{60}{360} \times 2\pi \times 3185 = \frac{1}{6} \times 2 \times 3.142 \times 3185 = \frac{1}{6} \times 20018.54 = 3336.42$ km [2]

Great circle distance: angular distance = $\cos^{-1}(\sin 60 \sin 60 + \cos 60 \cos 60 \cos 60) = \cos^{-1}(0.75 + 0.25 \times 0.5) = \cos^{-1}(0.875) \approx 28.955^\circ$ [3]

Distance = $\frac{28.955}{360} \times 2\pi \times 6370 = \frac{28.955}{360} \times 40038.68 \approx 3220.5$ km [2]

Time = distance/speed = $3220.5/800 \approx 4.03$ hours [1]

Alternative method/answer:

Great circle distance approx 3220 km, time approx 4.03 hours

- 24.** A particle moves along a straight line such that its velocity v m/s at time t seconds is given by $v = 3t^2 - 12t + 9$ for $t \geq 0$. (a) Find the initial velocity. (b) Find the time when the particle is momentarily at rest. (c) Find the displacement of the particle from its starting point after 3 seconds. (d) Find the total distance traveled in the first 3 seconds. (10 marks)

Initial velocity: $v(0) = 3(0)^2 - 12(0) + 9 = 9$ m/s [2]

At rest when $v=0$: $3t^2 - 12t + 9 = 0 \Rightarrow t^2 - 4t + 3 = 0 \Rightarrow (t-1)(t-3) = 0$