

EDUZEDCO
121/2 MATHEMATICS
Paper 2
Form 4 End Term Examination

Time: 2 ½ Hours

Name:

Adm No:

School:

Class:

Signature:

Date:

INSTRUCTIONS TO CANDIDATES

- (a) Write your name and index number in the spaces provided at the top of this page.
- (b) Write your school name, sign and write the date of the examination in the spaces provided above.
- (c) This paper consists of **Two** sections: **Section I** and **Section II**.
- (d) Answer **ALL** questions in Section I and any **five questions** from Section II.
- (e) Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question.
- (f) Marks may be given for correct working even if the answer is wrong.
- (g) Non-Programmable silent electronic calculators and KNEC Mathematical Tables may be used.
- (h) This paper consists of **8 printed pages**.
- (i) Candidates should check the question paper to ensure that all the pages are printed as indicated and no questions are missing.
- (j) Candidates should answer the questions in English.

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

SECTION II

17	18	19	20	21	22	23	24	Total	Grand Total

SECTION I (50 Marks)

Answer **all** questions in this section in the spaces provided.

1. Make x the subject of the formula $y = \frac{ax + b}{c}$. (2 mks)

2. Simplify $\frac{\sqrt{5}}{\sqrt{2}}$, leaving your answer in surd form. (3 mks)

3. The length of a rectangle is measured as 12.5 cm to the nearest 0.1 cm. Find the percentage error in the length. (3 mks)

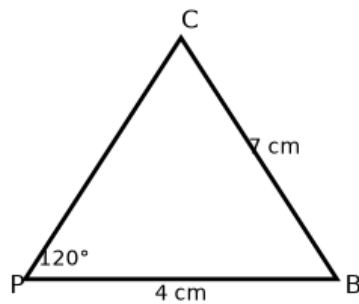
4. y varies jointly as x and z . If $y = 20$ when $x = 2$ and $z = 5$, find y when $x = 3$ and $z = 4$. (3 mks)

5. The third term of a geometric progression is 12 and the sixth term is 96. Find the common ratio. (3 mks)

6. Express $x^2 + 6x + 5$ in the form $(x + p)^2 + q$. (3 mks)

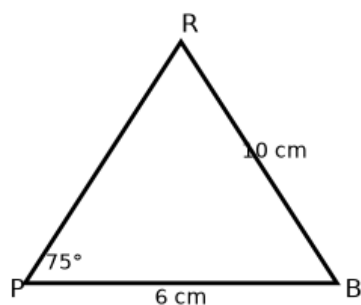
7. Use the binomial expansion to expand $(1 + 2x)^4$ up to the term in x^3 . (3 mks)

8. In the diagram below, O is the centre of the circle, AB is a chord, and OT is perpendicular to AB at T . If $AB = 10$ cm and $OT = 4$ cm, find the radius of the circle. (3 mks)



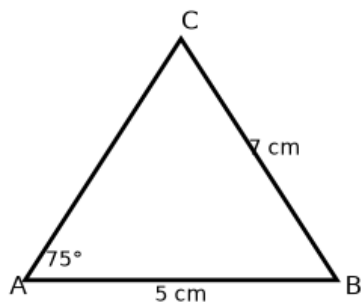
9. Solve the equation $2\sin\theta + 1 = 0$ for $0^\circ \leq \theta < 360^\circ$. (3 mks)

10. Given vectors (3 mks)
 $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and
 $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$, find
 $|\mathbf{a} - \mathbf{b}|$.



11. A transformation (4 mks)
 T is
represented by the matrix
 $\begin{pmatrix} 2 & 11 & 2 \\ 2 & 2 & 2 \end{pmatrix}$

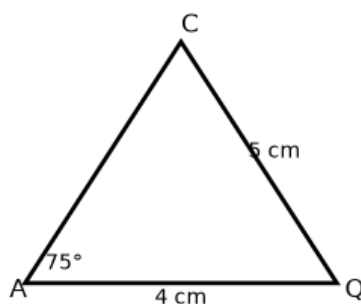
. Find the image of the point
 $(1, -2)$ under
 T .



12. A bag contains 3 red balls and 5 blue balls. Two balls are drawn without replacement. Find the probability that both balls are red. (4 mks)

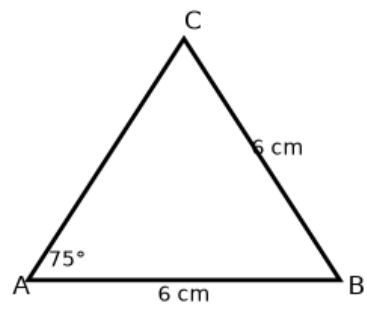
13. Evaluate $\int_1^2 (3x^2 - 2x) dx$. (4 mks)

14. The diagram below shows a rectangular prism $ABCDEFGH$ (4 mks)
 $AB = 6$ cm,
 $BC = 4$ cm, and
 $CG = 3$ cm. Find
the angle between the line
 AG and the
base $ABCD$.



15. Two points on the equator have longitudes (4 mks)
 30° E and
 60° E. Find the
distance between them along the equator, taking the Earth's radius as
6370 km. Give
your answer to the nearest km.

16. Construct the locus of a point (4 mks)
 P that moves
such that P
is equidistant from two fixed points
 A and
 B . Describe
the locus.



SECTION II (50 Marks)

Answer any **five** questions from this section in the spaces provided.

17. In a certain country, income tax is charged as follows: first \ (10 mks)

\$10,000 tax-free, next \$ 20,000 at

10%, next \ \$30,000 at 20%, and all income above \$
60,000 at 30%. Calculate the tax payable on an annual income of \ \$80,000.

Calculate the tax on the first \ \$10,000.

Calculate the tax on the next \ \$20,000.

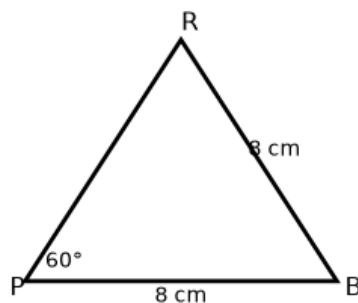
Calculate the tax on the next \ \$30,000.

Calculate the tax on the remaining income.

Find the total tax payable.

18. The marks of 40 students in a test are given below: Marks: 0-10, 10-20, (10 mks)

20-30, 30-40, 40-50 Frequency: 4, 8, 12, 10, 6 (a) Construct a cumulative
frequency table. (b) Draw a cumulative frequency curve (ogive). (c) Estimate
the median mark.



Construct a cumulative frequency table.

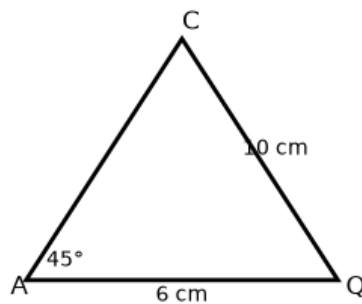
Draw a cumulative frequency curve.

Estimate the median mark.

19. A point $P(2, 3)$ is (10 mks)
transformed by
 $T_1 =$
 $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

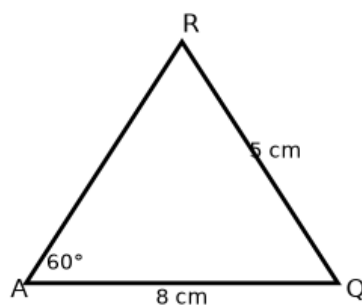
Followed by $T_2 = \begin{pmatrix} 0 & 5 \\ 1 & 1 \end{pmatrix}$

. Find the image of P under the combined transformation.



Find the image after T_1
Apply T_2 to the result.
Write the final coordinates.

20. In the diagram, O is the centre of the circle. (10 mks)
 $\angle AOB = 80^\circ$
 $\angle BOC = 70^\circ$
 $\angle ABC$ and . Find .

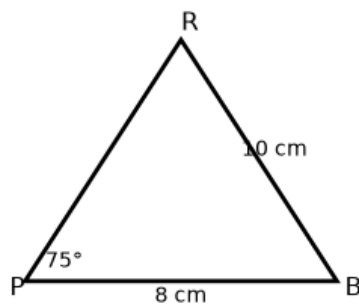


Find $\angle AOC$.

Calculate $\angle ABC$

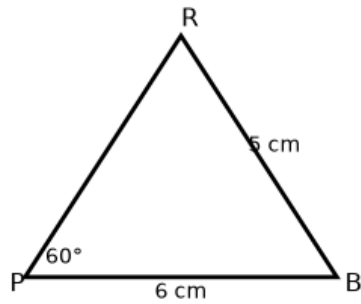
using circle theorems.

21. Points P , Q , and R have position vectors
 $\mathbf{p} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$,
 $\mathbf{q} = 4\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$, and
 $\mathbf{r} = 8\mathbf{i} + 9\mathbf{j} + 8\mathbf{k}$ respectively. Show that P , Q , and R are collinear and find the ratio $PQ : QR$.



Find vectors $\vec{PQ}$ and $\vec{QR}$.
 Show they are parallel.
 Find the ratio of their magnitudes.

22. Sketch the graph of $y = 3 \sin(2x)$ for $0 \leq x \leq 180^\circ$.
 Indicate the amplitude and period.



State the amplitude.

State the period.

Sketch the graph.

Label key points.

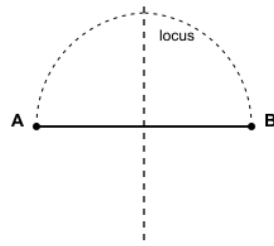
23. A plane flies from point $A(30^\circ N, 20^\circ E)$ to point $B(30^\circ N, 40^\circ W)$ along a parallel of latitude. Calculate the distance in km, given that the Earth's radius is 6370 km. (10 mks)

Find the angular difference in longitude.

Convert to radians.

Calculate the distance along the parallel.

24. A particle moves along a straight line such that its velocity v m/s at time t seconds is given by $v = 3t^2 - 2t + 1$. (10 mks)
Find: (a) The displacement after 2 seconds, given that initial displacement is 0. (b) The acceleration after 1 second.



Find the displacement function by integration.

Evaluate displacement at $t=2$.

Find acceleration by differentiation.

Evaluate acceleration at $t=1$.

121/2 MATHEMATICS MARKING SCHEME

Form 4 Paper 2

SECTION I (50 marks)

1. Make x the subject of the formula $y = \frac{ax + b}{c}$. (2 marks)

Multiply both sides by c : $cy = ax + b$ [1]

Subtract b : $cy - b = ax$, then divide by a : $x = (cy - b)/a$ [1]

2. Simplify $\frac{3}{\sqrt{5} - \sqrt{2}}$, leaving your answer in surd form. (3 marks)

Multiply numerator and denominator by conjugate: $(3(\sqrt{5} + \sqrt{2})) / ((\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}))$ [1]

Simplify denominator: $(\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3$ [1]

Simplify: $(3(\sqrt{5} + \sqrt{2}))/3 = \sqrt{5} + \sqrt{2}$ [1]

3. The length of a rectangle is measured as 12.5 cm to the nearest 0.1 cm. Find the percentage error in the length. (3 marks)

Absolute error = 0.05 cm [1]

Percentage error = $(0.05/12.5) \times 100\%$ [1]

= 0.4% [1]

4. y varies jointly as x and z . If $y = 20$ when $x = 2$ and $z = 5$, find y when $x = 3$ and $z = 4$. (3 marks)

$y = kxz$, substitute: $20 = k(2)(5) \Rightarrow k = 2$ [1]

$y = 2 \times 3 \times 4$ [1]

$y = 24$ [1]

5. The third term of a geometric progression is 12 and the sixth term is 96 . Find the common ratio. (3 marks)

$ar^2 = 12$, $ar^5 = 96$ [1]

Divide: $(ar^5)/(ar^2) = 96/12 \Rightarrow r^3 = 8$ [1]

$r = 2$ [1]

6. Express $x^2 + 6x + 5$ in the form $(x + p)^2 + q$. (3 marks)

Half of 6 is 3, so $(x+3)^2 = x^2 + 6x + 9$ [1]

$x^2 + 6x + 5 = (x+3)^2 - 9 + 5$ [1]

= $(x+3)^2 - 4$ [1]

7. Use the binomial expansion to expand $(1 + 2x)^4$ up to the term in x^3 . (3 marks)

$1^4 + 4(1^3)(2x) + 6(1^2)(2x)^2 + 4(1)(2x)^3$ [1]

= $1 + 8x + 24x^2 + 32x^3$ [2]

8. In the diagram below, O is the centre of the circle, AB is a chord, and OT is perpendicular to AB at T . If $AB = 10$ cm and $OT = 4$ cm, find the radius of the circle. (3 marks)

$AT = AB/2 = 5$ cm [1]

In right triangle OAT : $R^2 = OT^2 + AT^2 = 4^2 + 5^2 = 16 + 25 = 41$ [1]

$R = \sqrt{41}$ cm [1]

9. Solve the equation $2 \sin \theta + 1 = 0$ for $0^\circ \leq \theta \leq 360^\circ$. (3 marks)

$2 \sin \theta = -1 \Rightarrow \sin \theta = -1/2$ [1]

Reference angle = 30° , $\sin$ negative in QIII and QIV [1]

$$\theta = 180 + 30 = 210^\circ, 360 - 30 = 330^\circ \text{ [1]}$$

- 10.** Given vectors $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$, find $|\mathbf{a} - \mathbf{b}|$. (3 marks)

$$\mathbf{a} - \mathbf{b} = (2-1)\mathbf{i} + (-1-2)\mathbf{j} + (3-(-1))\mathbf{k} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k} \text{ [1]}$$

$$|\mathbf{a} - \mathbf{b}| = \sqrt{1^2 + (-3)^2 + 4^2} = \sqrt{1+9+16} = \sqrt{26} \text{ [2]}$$

- 11.** A transformation T is represented by the matrix $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$. Find the image of the point $(1, -2)$ under T . (4 marks)

$$\text{Multiply matrix by vector: } (2 \times 1 + 1 \times (-2), 3 \times 1 + 2 \times (-2)) \text{ [2]}$$

$$= (2-2, 3-4) = (0, -1) \text{ [2]}$$

- 12.** A bag contains 3 red balls and 5 blue balls. Two balls are drawn without replacement. Find the probability that both balls are red. (4 marks)

$$P(\text{first red}) = 3/8 \text{ [1]}$$

$$P(\text{second red} \mid \text{first red}) = 2/7 \text{ [1]}$$

$$P(\text{both red}) = (3/8) \times (2/7) = 6/56 \text{ [1]}$$

$$= 3/28 \text{ [1]}$$

- 13.** Evaluate $\int_1^2 (3x^2 - 2x) \, dx$. (4 marks)

$$\text{Integrate: } \int (3x^2 - 2x) \, dx = x^3 - x^2 \text{ [2]}$$

$$\text{Evaluate: } (2^3 - 2^2) - (1^3 - 1^2) = (8-4) - (1-1) = 4 - 0 = 4 \text{ [2]}$$

- 14.** The diagram below shows a rectangular prism $ABCDEFGH$ with $AB = 6$ cm, $BC = 4$ cm, and $CG = 3$ cm. Find the angle between the line AG and the base $ABCD$. (4 marks)

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13} \text{ cm [1]}$$

$$AG = \sqrt{AC^2 + CG^2} = \sqrt{52 + 9} = \sqrt{61} \text{ cm [1]}$$

$$\text{Angle between AG and base is angle AGC? Actually angle between AG and AC (since AC in base). So } \cos \theta = AC/AG = \sqrt{52}/\sqrt{61} \text{ [1]}$$

$$\theta = \cos^{-1}(\sqrt{52}/\sqrt{61}) \approx 24.0^\circ \text{ [1]}$$

- 15.** Two points on the equator have longitudes 30°E and 60°E . Find the distance between them along the equator, taking the Earth's radius as 6370 km. Give your answer to the nearest km. (4 marks)

$$\text{Angular difference} = 60^\circ - 30^\circ = 30^\circ \text{ [1]}$$

$$\text{Convert to radians: } 30^\circ \times \pi/180 = \pi/6 \text{ rad [1]}$$

$$\text{Distance} = \text{radius} \times \text{angle} = 6370 \times \pi/6 \text{ [1]}$$

$$\approx 3335 \text{ km (to nearest km) [1]}$$

- 16.** Construct the locus of a point P that moves such that P is equidistant from two fixed points A and B . Describe the locus. (4 marks)

$$\text{Draw points A and B [1]}$$

$$\text{Draw line AB and construct its perpendicular bisector [2]}$$

$$\text{Locus is the perpendicular bisector of AB [1]}$$

SECTION II (50 marks)

- 17.** In a certain country, income tax is charged as follows: first $\$10,000$ tax-free, next $\$20,000$ at 10%, next $\$30,000$ at 20%, and all income above $\$60,000$ at 30%. Calculate the tax payable on an annual income of $\$80,000$. (10 marks)

First \$10,000: tax = \$0 [2]

Next \$20,000: tax = 10% of 20,000 = \$2,000 [2]

Next \$30,000: tax = 20% of 30,000 = \$6,000 [2]

Remaining income: 80,000 - 60,000 = \$20,000; tax = 30% of 20,000 = \$6,000 [2]

Total tax = 0+2000+6000+6000 = \$14,000 [2]

- 18.** The marks of 40 students in a test are given below: Marks: 0-10, 10-20, 20-30, 30-40, 40-50 Frequency: 4, 8, 12, 10, 6 (a) Construct a cumulative frequency table. (b) Draw a cumulative frequency curve (ogive). (c) Estimate the median mark. (10 marks)

Cumulative frequencies: 4, 12, 24, 34, 40 [3]

Plot points (10,4), (20,12), (30,24), (40,34), (50,40) and join with smooth curve [4]

Median: from graph, at cumulative frequency 20, read mark ≈ 27.5 [3]

- 19.** A point $P(2, 3)$ is transformed by $T_1 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ followed by $T_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Find the image of P under the combined transformation. (10 marks)

After T_1 : $(1 \cdot 2 + 2 \cdot 3, 3 \cdot 2 + 4 \cdot 3) = (2+6, 6+12) = (8, 18)$ [4]

After T_2 : $(0 \cdot 8 + 1 \cdot 18, 1 \cdot 8 + 0 \cdot 18) = (18, 8)$ [4]

Final image: $(18, 8)$ [2]

- 20.** In the diagram, O is the centre of the circle. $\angle AOB = 80^\circ$ and $\angle BOC = 70^\circ$. Find $\angle ABC$. (10 marks)

$\angle AOC = 80^\circ + 70^\circ = 150^\circ$ [3]

Angle at centre is twice angle at circumference: $\angle ABC = \frac{1}{2} \angle AOC$ (reflex? Actually angle subtended by arc AC). But careful: $\angle ABC$ subtends arc AC (the minor arc? Actually points A, B, C on circle, angle ABC is angle at B. The central angle for arc AC is $\angle AOC = 150^\circ$. So $\angle ABC = \frac{1}{2} \times 150^\circ = 75^\circ$. [7]

- 21.** Points P , Q , and R have position vectors $\mathbf{p} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\mathbf{q} = 4\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$, and $\mathbf{r} = 8\mathbf{i} + 9\mathbf{j} + 8\mathbf{k}$ respectively. Show that P , Q , and R are collinear and find the ratio $PQ:QR$. (10 marks)

$PQ = q - p = (2i+2j+3k)$, $QR = r - q = (4i+4j+6k)$ [3]

$QR = 2PQ$, so vectors are parallel and P , Q , R collinear [4]

Ratio $PQ:QR = 1:2$ (since $QR = 2PQ$) [3]

- 22.** Sketch the graph of $y = 3 \sin(2x)$ for $0^\circ \leq x \leq 180^\circ$. Indicate the amplitude and period. (10 marks)

Amplitude = 3 [2]

Period = $360/2 = 180^\circ$ [2]

Sketch: sine wave from 0 to 180° , starting at 0, max at 45° (value 3), zero at 90° , min at 135° (value -3), zero at 180° [4]

Label key points: $(0,0)$, $(45,3)$, $(90,0)$, $(135,-3)$, $(180,0)$ [2]

- 23.** A plane flies from point $A(30^\circ \text{ N}, 20^\circ \text{ E})$ to point $B(30^\circ \text{ N}, 40^\circ \text{ W})$ along a parallel of latitude. Calculate the distance in km, given that the Earth's radius is 6370 km. (10 marks)

Angular difference = 20° E to $40^\circ \text{ W} = 60^\circ$ [2]

Convert to radians: $60^\circ \times \pi/180 = \pi/3$ rad [2]

Distance along parallel = $R \cos(\text{latitude}) \times \text{angle} = 6370 \times \cos(30^\circ) \times \pi/3$ [4]

$= 6370 \times (\sqrt{3}/2) \times \pi/3 \approx 5778$ km [2]

24. A particle moves along a straight line such that its velocity v m/s at time t seconds is given by $v = 3t^2 - 2t + 1$. Find: (a) The displacement after 2 seconds, given that initial displacement is 0. (b) The acceleration after 1 second. (10 marks)

$$\text{Displacement } s = \int v \, dt = \int (3t^2 - 2t + 1) dt = t^3 - t^2 + t + C; \text{ at } t=0, s=0 \Rightarrow C=0 \quad [3]$$

$$s(2) = 8 - 4 + 2 = 6 \text{ m} \quad [2]$$

$$\text{Acceleration } a = dv/dt = 6t - 2 \quad [3]$$

$$a(1) = 6(1) - 2 = 4 \text{ m/s}^2 \quad [2]$$